Department of Mathematics
Panjab University, Chandigarh

Schedule of Two Days National Conference on Algebra
February 23, 24, 2018

DAY 1

Registration: 9.00 - 9.30

Inauguration and Tea: 9.30 - 11.00

Lecture 1; 11.00 - 12.00: Speaker: Kapil Hari Paranjape, IISER Mohali

Title: *A question related to Complex Multiplication*

Abstract: Within the broad framework of (pure) Hodge structures is the arithmetically interesting class of Hodge structures with complex multiplication. The (generalised) Hodge conjecture throws up some intriguing questions about such Hodge structures. Nori has recently pointed out some of these. We will discuss these questions and their possible solutions.

Lecture 2; 12.00 - 13.00: Speaker: J. K. Verma, IIT Bombay

Title: *The tight Hilbert polynomial of an ideal*

Abstract: Let $(R, \mathfrak{m})$ be an analytically unramified local ring of dimension d and characteristic p where p is a prime number.

Let I be an $\mathfrak{m}$-primary ideal of R. Let I^* denote the tight closure of I. We introduce the *tight Hilbert function* $\ell(R/(I^n)^*)$ and show that it is given by a polynomial of degree d for large n. We call this as the *tight Hilbert polynomial of* I. We show that F-rationality of R can be characterised in terms of the coefficients of the tight Hilbert polynomial of an ideal generated by a system of parameters in R. We calculate the tight Hilbert polynomial of certain ideals in 2-dimensional hyper-surface rings and Stanley-Reisner rings of any dimension.

This is Joint work with Kriti Goel (IIT Bombay) and Vivek Mukundan (University of Virginia)

Lunch: 13.00 - 14.30

Lecture 3; 14.30 - 15.30: Speaker: Sudhir Ghorpade, IIT Bombay

Title: *Vanishing ideals, Grobner bases and the number of points of varieties over finite fields*

Abstract: It is an elementary fact if k is an infinite field and if a polynomial in m variables with coefficients in k vanishes at every point of the affine m-space over k, then it is the zero polynomial. Likewise if a homogeneous polynomial in $m+1$ variables with coefficients in k vanishes at every point of the projective m-space over k, then it is the zero polynomial. However, if k is a finite field, then there are many nonzero polynomials that vanish at every element of the affine or the projective space, and these give rise to the vanishing ideals in the corresponding polynomial rings over k. An explicit determination of the vanishing ideal of the affine m-space over $\mathbb{F}_q$ is classical and goes back at least to Terjanian (1966) in connection with his work on Chevalley-Warning theorem. Explicit determination of the vanishing ideal of the projective m-space over $\mathbb{F}_q$ is relatively recent and due to Mercier and Rolland (1998).

We will review these facts and also discuss Gröbner bases for these vanishing ideals. Further, we will discuss how these things can be used for determining or estimating the number of points of algebraic

varieties (that are not necessarily irreducible) over finite fields. This is done using the so-called "footprint bound", which is quite well-understood and widely utilized in the affine case. The projective case is relatively less explored and has some peculiarities. Nonetheless, we will show how a "projective footprint bound" can be obtained, and how it can be applied to deduce an inequality due to Serre about the maximum number of $\mathbb{F}_q$-rational points that a projective hypersurface of a given degree can have.

Parts of this talk are based on a joint work with Peter Beelen and Mrinmoy Datta.

Tea: 15.30 - 16.00

Lecture 4; 16.00 - 17.00: Speaker: Ananthnarayan Hariharan, IIT Bombay

Title: *Boij-Soderberg theory over standard graded rings*

Abstract: We begin with a quick introduction to the notion of Betti numbers over local or graded rings, and some problems related to them. Most of the talk will focus on the graded case. We will discuss the motivation behind the Boij-Soderberg conjectures (2008, followed by a quick word on the techniques used in their resolution by Eisenbud-Schreyer (2009). In the rest of the talk, we will see what works more generally, and try to identify the difficulties in applying the same techniques in other cases.

The last part of this talk is joint work with Rajiv Kumar.

DAY 2

Lecture 1; 10.00 - 11.00: Speaker: Dipendra Prasad, TIFR Mumbai

Title: *On the size of representations of finite and infinite groups (of Lie type)*

Abstract: The dimension of an irreducible representation tells a lot about the representation. In this talk we will discuss what are these possible dimensions, and also how these notions can also be defined both for infinite dimensional representations of real and p-adic Lie groups, and play a major role in the subject.

Tea: 11.00 - 11.30

Lecture 2; 11.30 - 12.30: Speaker: Pooja Singla, IISc Bangalore

Title: *On Irreducible representations of SL_2 over complete discrete valuation rings of even characteristic*

Abstract: Let R be complete discrete valuation ring such that residue field has characteristic p. In this talk we will focus on the construction of complex continuous irreducible representations of General Linear group $GL_2(R)$ and its subgroup Special Linear group $SL_2(R)$.

The construction of all irreducible representations of $GL_2(R)$ has already appeared in the work of Kutzko and Stasinski. Similarly, construction of all irreducible representations of $SL_2(R)$ for p odd has also appeared in the work of Zaikin-Japirain. However for the case $p = 2$, a construction of irreducible representations of $SL_2(R)$ is not yet known for general R.

For $R = \mathbb{Z}_2$ (the ring of 2-adic integers) a construction of the irreducible representations of $SL_2(R)$ was obtained by Nobs. In this talk we will describe a method to construct certain class of irreducible representations of $SL_2(R)$ for the case where $p = 2$ and R has positive characteristic. As an application, we are able to show that the complex group algebras of $SL_2(\mathbb{Z}/2^m\mathbb{Z})$ and $SL_2(F_2[t]/t^m)$ are not isomorphic for any even $m > 2$.

This is based on ongoing joint work with Hassain M.

Lecture 3; 12.30 - 13.00: Speaker: Gurleen Kaur, Panjab University, Chandigarh

Title: *Simple components of rational group algebra*

Abstract: A fundamental problem in group algebras is to determine the structure of simple components of rational group algebra. The problem is of great interest due to its applications in various other problems in group rings, for example in the study of unit group of integral group ring, the isomorphism problem for group algebras and the group codes. In this talk, we describe the structure of simple components of rational group algebra for a large class of monomial groups including abelian-by-supersolvable and subnormally monomial groups. This description is given using the subgroup structure of the group.

Lunch: 13.00 - 14.30

Lecture 4; 14.30 - 15.30: Speaker: Prabal Paul, Bits-Pilani Goa

Title: *On some recent trends in Algebraic Cryptography*

Abstract: Secret Sharing or Sharing a secret among many participants is an integral part of Algebraic Cryptography. Algebraic Coding theory is very much useful in Secret Sharing. In fact, Maximum Distance Separable (MDS) codes are very much useful in Secret Sharing. To hide a secret, it is natural to use a codeword of an MDS code. In this talk, we address some such Mathematical challenges for codes over finite fields, where the characteristic p of the finite field is a large prime number.

Tea: 15.30-16.00

Lecture 5; 16.00 - 16.30: Speaker: Sugandha Maheshwary, IISER Mohali

Title: *Algebra with GAP: Mathematical and computational aspects*

Abstract: In this talk, I shall give an introduction to GAP- which is a system for computational discrete algebra. I shall give an overview of how GAP can be used in research and teaching for studying groups and their representations, rings, vector spaces, algebras, combinatorial structures, and more. Recent contribution to the package "Wedderga", meant to compute the Wedderburn decomposition of group algebras, shall also be discussed.

Lecture 6; 16.30 - 17.00: Speaker: Leena Jinda, Panjab University, Chandigarh

Title: *p-Invariant and Witt Classes of Fields*

Abstract: Let F be a field of characteristic $\neq 2$ with finitely many square classes. Quaternion algebras over a field F are four dimensional simple associative algebras that whose centre equals F. In this talk we define a new invariant, that we call p-invariant, of fields taking values in positive rationals. In a certain sense, that we shall elaborate in the talk, p-invariant determines the probability that a randomly picked quaternion algebra over F is a division algebra. We compute p-invariant for some well known fields and establish that it can be used to count distinct Witt classes of fields.. In our main theorem we conclude that if the p-invariant of a field F with q_0 many square classes is p_0 then the p-invariant of $F((t_1))((t_2))\dots((t_n))$ will be equal to $1 - \frac{1-p_0}{4^n} - \frac{3}{2^n q_0}\left(1 - \frac{1}{2^n}\right)$.