

**PANJAB UNIVERSITY CHANDIGARH- 160 014 (INDIA)**

(Estd. under the Panjab University Act VII of 1947-enacted by the Govt. of India)



**FACULTY OF SCIENCE**

**SYLLABI**

**FOR**

**M.Sc. (HONOURS SCHOOL) MATHEMATICS**

**1<sup>ST</sup> TO 4<sup>th</sup> SEMESTER**

**EXAMINATIONS 2016-17**

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**OUTLINES OF TESTS, SYLLABI AND COURSES OF READING FOR M.SC.  
(HONS. SCHOOL) IN MATHEMATICS FOR SEMESTER I AND SEMESTER II  
EXAMINATIONS 2016-2017**

**Outlines of Tests**

1. Every student will have to take five papers.
2. Each paper/course shall carry 100 marks.
3. The duration of the examination shall be of three hours.
4. The question paper will have two parts each having four questions. Candidates will attempt five questions in all choosing atleast two from each part.
5. All questions carry equal marks

**Semester I**

Every student will have to take five papers given below:

Paper I :    Math 701S – Topology  
                  OR  
                  Math 702S – Real Analysis

Paper II:    Math 703S - Topics in Algebra-I  
                  OR  
                  Math 704S – Groups and Rings

Paper III:    Math 705S - Linear Programming  
                  OR  
                  Math 706S - Number Theory-I

Paper IV:    Math 707S – Complex Analysis-I

Paper V:    Math 708S – Classical Mechanics-I

**The above mentioned courses will be offered to the students depending upon their background.**

## **Semester-II**

- Paper I:    Math 721S -        Functional Analysis  
                          OR  
                 Math 722S -    Measure Theory
- Paper II:    Math 723S -        Topics in Algebra-II  
                          OR  
                 Math 724S -        Modules & Fields
- Paper III:    Math 725S -        Non-Linear Programming  
                          OR  
                 Math 726S -        Number Theory-II
- Paper IV:    Math 727S -        Complex Analysis-II
- Paper V:    Math 728S -        Classical Mechanics-II

**The students who have studied Courses – MATH 701S, MATH 703S, MATH 705S in Semester I will have to take MATH 721S, MATH 723S, MATH 725S in Semester II.**

**Students who have studied MATH 702S, MATH 704S, MATH 706S in Semester I will have to take MATH 722S, MATH 724S, MATH 726S in Semester II**

## Semester-I

### Paper-I

#### **MATH-701S: Topology**

[7 hrs per week(including tutorials)]

[Max.Marks:100]

(Final-80+Internal Assessment-20)

Time : 3hrs

#### **Objectives**

*The course is an introductory course on point-set topology so as to enable the reader to understand further deeper topics in topology like Differential/ Algebraic Topologies etc.*

- Note :**
- 1. The question paper will have eight questions. Candidates will attempt five questions.**
  - 2. There will be four questions from each part and the students will be required to answer atleast two questions from each part.**
  - 3. All questions carry equal marks.**

#### **PART – I**

Topological Spaces, bases for a topology, the order topology, the product topology on  $X \times Y$ , the subspace topology, closed sets and limit points, continuous functions, the product topology, the metric topology, the quotient topology.

[Scope as in the relevant sections in Chapter 2 of the book 'Topology', second edition 2002, by James R. Munkres.]

Connected spaces, connected subspaces of the real line, components and local connectedness

#### **PART –II**

Compact spaces, compact space of the real line, limit point compactness, local compactness, nets.

[Scope as in the relevant sections in Chapter 3 of the book 'Topology', second edition 2002, by James R. Munkres.]

The countability axioms, the separation axioms, normal spaces, the Urysohn Lemma, the Urysohn Metrization Theorem, the Tietze Extension Theorem, the Tychonoff Theorem.

[Scope as in the relevant sections in Chapters 4 and 5 of the book 'Topology', second edition 2002, by James R. Munkres.]

**References:**

1. James R. Munkers : Topology, 2<sup>nd</sup> Edition, 2002, Prentice Hall of India.
2. James Dugundji : Topology, UBS Publishers, 1<sup>st</sup> Edition, 1990. .
3. John L. Kelley : General Topology (Van Nostrand), 1<sup>st</sup> Edition, 1955.
4. Bourbaki – General Topology (Reading, Addison-Wesley), 1<sup>st</sup> Edition, 1989.
5. G.G. Simmons - Introduction to Topology and Modern Analysis Tokyo, McGraw Hill, Kongakusha), 1<sup>st</sup> Edition, 1963.
6. W.J. Thron, - Topological structures (N.Y.Holt) (Scope as in Chapters IV to XV, Chapter XVI: def. 16.4 and results including Tychonoff's theorem and Chapter XVIII of the reference 4), 1<sup>st</sup> Edition, 1966..
7. E.T. Copson – Metric Spaces (Cambridge University Press), 1<sup>st</sup> Edition, 1963.
8. S. Willord – General Topology (Addison Wesley Publishing Company), 1970.

**OR****MATH 702S: Real Analysis**

[7 hrs per week(including tutorials)]

[Max.Marks:100]

(Final-80+Internal Assessment-20)

Time : 3hrs

**Objective**

*The aim of this course is to make the students learn fundamental concepts of metric spaces, The Riemann-Stieltjes integral as a generalization of Riemann Integral, the calculus of several variables and basic theorem.*

- Note :**
1. **The question paper will have eight questions. Candidates will attempt five questions.**
  2. **There will be four questions from each part and the students will be required to answer atleast two questions from each part.**
  3. **All questions carry equal marks.**

**PART-I**

(i) Basic Topology: Finite, countable and uncountable sets, metric spaces, compact sets, perfect sets, connected sets.

(ii) Sequences and series: Convergent sequences, subsequences, Cauchy sequences(in metric spaces), completion of a metric space, absolute convergence, addition and multiplication of series, rearrangements of series of real and complex numbers.

(iii) Continuity: Limits of functions (in metric spaces), continuous functions, continuity and compactness, continuity and connectedness, monotonic functions.

(iv) The Riemann-Stieltjes integral: Definition and existence of the Riemann-Stieltjes integral, properties of the integral, integration of vector-valued functions, rectifiable curves.

## PART II

(v) Sequences and series of functions: Problem of interchange of limit processes for sequences of functions, Uniform convergence, Uniform convergence and continuity, Uniform convergence and integration, Uniform convergence and differentiation, equicontinuous families of functions, Stone Weierstrass Theorem.

(vi) Differentiation: Differentiation of vector-valued functions.

(vii) Functions of several variables: The space of linear transformations on  $\mathbb{R}^n$  to  $\mathbb{R}^m$  as a metric space. Differentiation of a vector-valued function of several variables. The Inverse function theorem. The implicit function theorem

### Scope

For items (i) to (vii) as in relevant sections of Chapters 2 to 7 and Chapter 9 of the book at Sr. No. 6 in the list of references.

### References:

1. Apostol, Tom. 'Mathematical Analysis – a modern approach to Advanced Calculus, Addison – Wesley Publishing Company, Inc. 1957. (Indian Edition by Narosa Publishing House New Delhi also available).
2. Bromwich, T.J.I.A., 'An introduction to the theory of infinite series. 2<sup>nd</sup> Edition (Revised with the assistance of T.M. Mac Robert). Macmillan and Co. Ltd., New York, 1955.
3. Goldberg, R.R.: Methods of Real Analysis, Oxford and IHB Publishing Company, New Delhi.
4. Knopp, K.: 'Theory and Applications of Infinite series', Blackie and Sons Ltd. London and Glasgow, 2<sup>nd</sup> Edition 1951 (Reprinted 1957).
5. Malik, S.C., Savita Arora: Mathematical Analysis, New Age International (P) Ltd, New Delhi, 3<sup>rd</sup> Edition, 2008.
6. Rudin, Walter: 'Principles of Mathematical Analysis'. 3<sup>rd</sup> Edition (International Student Edition) McGraw-Hill Inc. 1976.
7. Shanti Narayan, 'A Course of Mathematical Analysis', S.Chand and Co. Ltd., New Delhi, 12<sup>th</sup> Revised Edition 1986.
8. Titchmarsh, E.C.: The Theory of functions, 2<sup>nd</sup> Edition, The English Language Book Society and Oxford University Press 1961.

## Paper-II

### **MATH 703S : Topics in Algebra-I**

[7 hrs/per week (including Tutorials)]

[Max. Marks: 100]

(Final-80+Internal Assessment-20)

Time : 3hrs

#### **Objective**

*The objective of this course is to introduce the basic ideas of Field Theory and Galois Theory and to see its application to the solvability of polynomial equations by radicals. Answers to some classical problems of ancient Greeks regarding the ruler and compass constructions shall be obtained as a consequence of the development of the subject. This course also provides the foundation required for more advanced studies in Algebra. The aim is also to develop necessary prerequisites for Math 723S.*

- Note :**
- 1. The question paper will have eight questions. Candidates will attempt five questions.**
  - 2. There will be four questions from each part and the students will be required to answer at least two questions from each part.**
  - 3. All questions carry equal marks.**

### **PART-I**

#### **Field Theory:**

Fields, Examples, Algebraic and Transcendental elements. The degree of a field extension. Adjunction of roots. Splitting fields. Finite fields. Algebraically closed fields, Separable and purely inseparable extensions. Perfect fields, primitive elements, Lagrange's theorem on primitive elements. Normal extensions, Galois extensions, The fundamental theorem of Galois Theory.

### **PART-II**

Symmetric functions. Cyclotomic extensions. Cyclic extensions, Norms and traces. Quintic Equations and solvability by radicals.

Review of Rings and ring homomorphism, ideals, quotient rings, zero divisors, nilpotent elements, units, prime ideals and maximal ideals, Nilradical and Jacobson radical, operation on ideals, extension and contraction of ideals, Modules and module homomorphisms, submodule and quotient module, operation on submodules, direct sum and product, finitely generated modules, exact sequences, tensor product of modules, restriction and extension of scalars, exactness property of the tensor product, Algebras, tensor product of algebras.

**Suggested Books**

1. M.F. Atiyah and I.G. MacDonald: Introduction **to Commutative Algebra**. Levant Books, Indian Edition, 2007.
2. M. Artin , Algebra, Prentice Hall of India Pvt. 1994.
3. Stewart-I, Galois Theory (Capman & Hall (1973).
4. J.-P. Escofier, Galois Theory, Graduate texts in Mathematics, 204, Springer Verlag, 1<sup>st</sup> Edition, 2001.
5. M.F. Atiyah and I.G. MacDonald: Introduction **to Commutative Algebra**. Levant Books, Indian Edition, 2007.
6. O. Zariski and P. Samuel, **Commutative Algebra**, Vols I and II, Springer 1975.
7. I. S. Luthar and I. B. S. Passi, Algebra Vol.4, Field Theory, Narosa Publishing House 2004.

**OR**

**MATH-704S: Groups and Rings**

[7 hrs/per week (including Tutorials)]  
 [Max. Marks: 100]  
 (Final-80+Internal Assessment-20)  
 Time : 3hrs

**Objective**

*This course covers some advanced topics of Group Theory and Ring Theory, which are two most important branches of algebra.*

- Note :**
1. **The question paper will have eight questions. Candidates will attempt five questions.**
  2. **There will be four questions from each part and the students will be required to answer at least two questions from each part.**
  3. **All questions carry equal marks.**

**PART-I**

Review of basic property of Groups, Dihedral groups, Symmetric groups and their conjugacy classes, Simple groups and their examples. Simplicity of  $A_n$  ( $n \geq 5$ ), Sylow Theorems and their applications, Direct Products, finite Abelian Groups, Fundamental Theorem on Finite Abelian Groups, Normal and Subnormal Series, Derived Series, Composition Series, Solvable Groups, Zassenhaus Lemma, Scherier's refinement theorem and Jordan-Holder Theorem.

**PART-II**

Review of Rings, Zero Divisors, Nilpotent Elements and Idempotents, Matrices, Quaternions, Ring of endomorphisms, polynomial rings in many variables, Factorization of polynomials in one variable over a field. Unique factorization



domains. Gauss Lemma, Eisenstein's Irreducibility Criterion, Unique Factorization in  $R[x]$  where  $R$  is a Unique Factorization Domain. Euclidean and Principal ideal domains.

### **Suggested Books**

1. David S. Dummit and Richard M Foote: Abstract Algebra, John Wiley & Sons, 2004.
2. I. N. Herstein: Topics in Algebra, 2<sup>nd</sup> Edition, Vikas Publishing House, New Delhi, 1976.
3. C. Musili: Rings and Modules, 2<sup>nd</sup> Revised Edition, Narosa Publishing House, New Delhi, 1994.
4. M. Artin: Algebra, Prentice Hall of India, New Delhi, 1994.
5. W. Burnside: The Theory of Groups of Finite Order, 2<sup>nd</sup> Edition, Dover, New York, 1955.
6. P.B. Bhattacharya, S.K. Jain and S.R. Nagpal: Basic Abstract Algebra, 2<sup>nd</sup> Edition, Cambridge University Press, 2002.
7. J. B. Fraleigh: A First Course in Abstract Algebra, 3<sup>rd</sup> Edition, Narosa Publishing House, New Delhi, 3<sup>rd</sup> Edition, 2003.
8. J. A. Gallian: Contemporary Abstract Algebra, 4<sup>th</sup> Edition, Narosa Publishing House, New Delhi, 1998.
9. B. Hartley and T. O. Hawkes: Rings, Modules and Linear Algebra, Chapman and Hall, 1<sup>st</sup> Edition, 1970.
10. T. W. Hungerford: Algebra, Springer 1974.
11. D. S. Malik, J. N. Mordeson and M. K. Sen: Fundamentals of Abstract Algebra, McGraw-Hill, New York 1997.
12. Surjeet Singh and Q. Zameeruddin, Modern Algebra, 7<sup>th</sup> Edition, Vikas Publishing House, New Delhi, 1993.
13. I.S.Luthar and I.B.S.Passi, Algebra Vol. 2, Rings, Narosa Publishing House, 1999.

**Paper-III**

**MATH 705S :Linear Programming**

**[7 hrs per week (including tutorials)]**

**Max.Marks : 100**

**[Final-80+Internal Assessment-20]**

**Time: 3hrs.**

***Objective***

*The objective of this course is to acquaint the students with the concept of convex sets, their properties and various separation theorems so as to tackle with problems of optimization of functions of several variables over polyhedron and their duals. The results, methods and techniques contained in this paper are very well suited to the realistic problems in almost every area.*

- Note:**
- 1. The question paper will have eight questions. Candidates will attempt five questions.**
  - 2. There will be four questions from each part and the students will be required to answer at least two questions from each part.**
  - 3. All questions carry equal marks.**

**PART-I**

Linear Programming and examples, Convex Sets, Hyperplane, Open and Closed half-spaces, Feasible, Basic Feasible and Optimal Solutions, Extreme Point & graphical methods. Simplex method, Charnes-M method, Two phase method, Determination of Optimal solutions, unrestricted variables, Duality theory, Dual linear Programming Problems, fundamental properties of dual Problems, Complementary slackness, Unbounded solution in Primal. Dual Simplex Algorithm, Sensitivity analysis.

**PART-II**

Parametric Programming, Revised Simplex method, Transportation Problems, Balanced and unbalanced Transportation problems, U-V method, Paradox in Transportation problem, Assignment problems, Integer Programming problems: Pure and Mixed Integer Programming problems, 0-1 programming problem, Gomory's algorithm, Branch & Bound Technique. Travelling salesman problem [scope as in reference no 2].

[Scope as in Chapter 2-5; Chapter 7-9 of the reference no.1, chapter 4-6 of reference no3, chapter 5 of reference no2].

**References:**

1. G. Hadley: Linear Programming, Narosa Publishing House, 6<sup>th</sup> edition. 1995.
2. N.S. Kambo: Mathematical Programming Techniques, 1984, Affiliated East-West Press Pvt.Ltd. New Delhi, Madras (Reprint 2005) revised Edition.

3. Suresh Chandra, Jayadeva and Aparna Mehra: Numerical Optimization with Applications, Narosa Publishing House, 1<sup>st</sup> Edition, 2009.
4. S.M. Sinha: Mathematical Programming, Theory and Methods, 1<sup>st</sup> Edition, Elsevier, 2006.

**OR**

**MATH 706S:      Number Theory-I**

**[7 hrs per week (including tutorials)]**

**Max.Marks : 100**

**[Final-80+Internal Assessment-20]**

**Time: 3hrs.**

**Objective:**

*The aim of this course is to teach the students about the basics of Elementary Number Theory starting with primes, congruences, quadratic residues, primitive roots, arithmetic functions. Apart from teaching the theory, stress will be on solving problems.*

- Note :**
- 1. The question paper will consist of two parts containing four questions each. Candidates will attempt five questions.**
  - 2. There will be four questions from each part and the students will be required to answer at least two questions from each part.**
  - 3. All questions carry equal marks.**

### **PART-I**

Divisibility, Greatest common divisor, Euclidean algorithm, The Fundamental theorem of arithmetic, Congruences, Residue classes and reduced residue classes, Chinese remainder theorem, Fermat's little theorem, Wilson's theorem, Euler's theorem and its application to a cryptography, Arithmetic functions  $\phi(n)$ ,  $d(n)$ ,  $\sigma(n)$ ,  $\mu(n)$ , Mobius inversion formula, Greatest integer function.

### **PART-II**

Primitive roots and indices. Quadratic residues, Legendre symbol, Euler's criterion, Gauss's lemma, Quadratic reciprocity law, Jacobi symbol. Representation of an integer as a sum of two and four squares. Diophantine equations  $ax + by = c$ ,  $x^2 + y^2 = z^2$ ,  $x^4 + y^4 = z^2$ . Binary quadratic forms and equivalence of quadratic Forms. Perfect numbers, Mersenne primes and Fermat numbers, Farey fractions.

**References:**

1. G. H. Hardy and E. M. Wright – An Introduction to Theory of Numbers, Oxford University Press, 6th Ed , 2008.

2. I. Niven, H. S. Zuckerman and H. L. Montgomery – An Introduction to the Theory of Numbers, John Wiley and Sons, (Asia) 5th Ed., 2004.
3. H. Davenport - The Higher Arithmetic, Camb. Univ. Press, 7<sup>th</sup> edition, (1999)
4. David M. Burton – Elementary Number Theory, Tata McGraw Hill, 6<sup>th</sup> Edition, 2007.

### **Paper-IV**

#### **MATH 707S :Complex Analysis-I**

**[7 hrs per week (including tutorials)]**

**Max.Marks : 100**

**[Final-80+Internal Assessment-20]**

**Time: 3hrs.**

#### **Objective**

*The objective of the course is to provide foundation for other related branches of Mathematics. Most of the topics covered are widely applicable in Applied Mathematics and Engineering. Moreover, while designing the syllabus, the syllabus of UGC NET was also considered.*

- Note:**
- 1. The question paper will have eight questions. Candidates will attempt five questions.**
  - 2. There will be four questions from each part and the students will be required to answer at least two questions from each part.**
  - 3. All questions carry equal marks.**

#### **PART-I**

Complex plane, geometric representation of complex numbers, joint equation of circle and straight line, stereographic projection and the spherical representation of the extended complex plane. Topology on the complex plane, connected and simply connected sets. Complex valued functions and their continuity. Curves, connectivity through polygonal lines.

Analytic functions, Cauchy-Riemann equations, Harmonic functions and Harmonic conjugates.

Power series, exponential and trigonometric functions,  $\arg z$ ,  $\log z$ ,  $a^z$  and their continuous branches. Complex Integration, line integral, Cauchy's theorem for a rectangle, Cauchy's theorem in a disc.

(Scope as in "Theory of Functions of a Complex Variable" by Shanti Narain).

#### **PART-II**

Index of a point with respect to a closed curve, Cauchy's integral formula, higher derivatives, Morrrera's theorem, Liouville's theorem, fundamental theorem of Algebra, Maximum Modules principle, Schwarz Lemma. The general form of

Cauchy's theorem. Taylor series and Laurent series. Singularities, Cauchy's residue theorem and Calculus of residues

(Scope as in "Complex Analysis" by D. V. Ahlfors, Chapter 4, §1, 2 §4.1 to 4.5 and §5.1 and the book "Theory of Functions Complex Variable" by Shanti Narayan, and in "Theory of Functions Complex Variable" by Shanti Narain, Chapter 1, 2, §39-44 and §47-50, 53, 54 of Chapter 4, §59-64 of Chapter 5., §79-88 of Chapter 6, §111-113, §117-118 of Chapter 9 and Chapter 11)

### References :

1. Shanti Narayan: Theory of Functions of a Complex Variable, S. Chand and Co., 7<sup>th</sup> Edition, 1986.
2. D.V. Ahlfors,, Complex Analysis, 3<sup>rd</sup> Edition, 1979 (International student edition), McGraw-Hill International Book Company.
3. J.B. Conway: Function of One Complex Variable, 2<sup>nd</sup> Edition, 1978, Corr 4<sup>th</sup> Print 1986, Graduate Texts, Springer-Verlag, Indian edition by Narosa Publishing House, New Delhi.
4. E. T. Copson: An Introduction to the Theory of Functions of a Complex Variable, The English Language Book Society and Oxford University Press, 1985.
5. K. Knopp: Theory of Functions, (Translated by F. Bagemite) in Two Volumes, Dover Publications, Inc., New York, 1945, 1947.
6. T. Pati: Functions of a Complex Variable, Allahabad, Pothishala, 1971.
7. S. Saks and A. Zygmund: Analytic Functions (Translated by E. J. Scott) Poland, Warszawa, 1952.
8. R. Silverman: Introductory Complex Analysis, Prentice-Hall Inc., Englewood Cliffs, N.J., 1967.
9. J.V. Deshpande: Complex Analysis, Tata McGraw-Hill Publishing Company Ltd., 1989.
10. E.C. Titchmarsh: The Theory of Functions, the English Language Book Society and Oxford University Press, 2<sup>nd</sup> Edition, 1961.
11. Wolfgang Tutschke and Harkrishan L. Vasudeva: An Introduction to Complex Analysis, Classical and Modern Approaches, Chapman and Hall/CRC, 2005.
12. S. Ponnusamy: Foundations of Complex Analysis by Narosa Publishing House, New Delhi, 2<sup>nd</sup> Edition, 2005.

## Paper-V

### **MATH 708: Classical Mechanics-I**

[7 hrs per week (including tutorials)]

Max.Marks : 100

[Final-80+Internal Assessment-20]

Time: 3hrs.

#### **Objective**

*The objective of this paper is to introduce the concept of variation of a functional and variational techniques. The Calculus of variation helps a lot to understand the Lagrangian and Hamiltonian equations for dynamical systems. Variational principles and their applications are introduced at large*

- Note :**
- 1. The question paper will have eight questions. Candidates will attempt five questions.**
  - 2. There will be four questions from each part and the students will be required to answer at least two questions from each part.**
  - 3. All questions carry equal marks.**

### **PART-I**

**Calculus of variations:** Functional and their properties, Motivating problems of Calculus of variations, Shortest distance, minimum surface of revolution, Brachistochrone problem, Isoperimetric problems, Geodesics, Fundamental lemma of Calculus of Variations, Euler's equation for one dependent function and its generalization to (i) n dependent functions, (ii) higher order derivatives, Variational problems with moving boundaries, Variation under constraints, Variational methods of Rayleigh-Ritz and Galerkin.

**Lagrangian Mechanics:** Generalized coordinates, Constraints, Holonomic and non-holonomic systems, Scleronomic and Rheonomic systems, Generalized velocity, Generalized potential, Generalized force, D'Alembert's principle, Lagrange's equation, Velocity dependent Potentials and Dissipation function, Expression of Kinetic energy using generalized velocity, Non-uniqueness in the choice of Lagrangian.

### **PART-II**

**Lagrangian Mechanics:** Hamilton's principle, Principle of Least action, Derivation of Lagrange's equations from Hamilton's principle, Cyclic coordinates, Conjugate momentum, Conservation theorems.

**Hamiltonian Mechanics:** Legendre's transformation, Hamilton's equations, Routhian, Poisson Bracket, Jacobi identity for Poisson bracket, Poisson theorem, Hamilton's equation in Poisson bracket, Canonical Transformation, Hamilton-Jacobi equations, Method of Separation of variables, Action – Angle variables, Lagrange Bracket. Invariance of Lagrange Bracket under Canonical Transformations.

**References:**

1. L. Elsgolts: Differential equations and the calculus of variations, Mir Publication
2. H. Goldstein, C. Poole and J. Safko: Classical Mechanics, 3<sup>rd</sup> Edition, Addition Wesley (2002)
3. F. Chorlton: Text book of Dynamics, CBS Publishers, 2<sup>nd</sup> Edition, Reprint 2002.
4. F. Grantmacher: Lecture in analytical Mechanics, Mir Publication, 1975.

## Semester-II

### Paper-I

#### **MATH 721S: Functional Analysis**

[7 hrs per week (including tutorials)]

Max.Marks : 100

[Final-80+Internal Assessment-20]

Time: 3hrs.

#### **Objective**

*The objective of this course is to introduce Banach and Hilbert spaces. The various operators on Hilbert and Banach spaces are also included.*

**Note : 1.The question paper will have eight questions. Candidates will attempt five questions.**

**2. There will be four questions from each part and the students will be required to answer at least two questions from each part.**

**3. All questions carry equal marks.**

#### **PART-I**

Baire Category theorem and its applications.

[Scope as in relevant topics of Chapter I from Ref.2]

Normed Spaces, with examples of Function spaces  $L^p([a,b])$ ,  $C([a,b])$  and  $C^1([a,b])$ , Sequence Spaces  $l^p$ ,  $c$ ,  $c_0$ ,  $c_{00}$  Banach Spaces, Hahn Banach theorem, open mapping theorem, closed graph theorem, Banach Steinhaus theorem (uniform boundedness principle), [Scope as in relevant topics from Chapter 2 & 3 of Ref.6.]

#### **PART- II**

Geometry of Hilbert spaces: Inner product spaces, orthonormal sets, Approximation and optimization, Projections and Riesz Representation theorem.

Bounded Operators on Hilbert spaces: Bounded operators and adjoints; normal, unitary and self adjoint operators, Spectrum and Numerical Range.

[Scope as in Ch.VI & VII (§25-27.7) of the book 'Functional Analysis' by B.V.Limaye, 1996.]

#### **References:**

1. S.K. Berberian: Introduction to Hilbert Spaces, (N.Y. O.W.P.), 1996.
2. C. Goffman and G. Pedrick: First Course in Functional Analysis, N. Delhi Prentice Hall of India, 1983.
3. F.K. Riesz and Bela Sz Nagy: Functional Analysis, (N.Y., Wingar), 1990.
4. A.H.Siddiqui: Functional Analysis (Tata-McGraw Hill), 1987.
5. Walter Rudin: Real and Complex Analysis(McGraw-Hill) 3<sup>rd</sup> Edition, 1986.



6. B.V. Limaye: Functional Analysis (New Age International (P) Ltd.), 2<sup>nd</sup> Edition, 1996.
7. H.L. Royden: Real Analysis, Pearson Prentice Hall, Dorling Kindersley (P) Ltd India, 3<sup>rd</sup> Edition, 2007.

**OR**

**MATH 722S: Measure Theory**

[7 hrs per week (including tutorials)]

Max.Marks : 100

[Final-80+Internal Assessment-20]

Time: 3hrs.

**Objective**

*The objective of this course is to study Lebesgue measure as generalisation of lengths, Lebesgue integral, Fundamental Theorem of Calculus and  $L^p$  spaces.*

- Note :**
- 1. The question paper will have eight questions. Candidates will attempt five questions.**
  - 2. There will be four questions from each part and the students will be required to answer at least two questions from each part.**
  - 3. All questions carry equal marks.**

**PART-I**

- (i) Lebesgue measure: Introduction, outer measure, measurable sets and Lebesgue measure, a non-measurable set, measurable functions, Littlewood's three principles.
- (ii) The Lebesgue Integral: The Lebesgue integral of a bounded function over a set of finite measure, the integral of a non-negative function, the general Lebesgue integral, convergence in measure.

**PART-II**

- (iii) Differentiation and Integration: Differentiation of monotone functions, differentiation of an integral, absolute continuity, convex functions .
- (iv) The Classical Banach spaces: The  $L^p$  spaces, Minkowski's and Holder's inequality, convergence and completeness.

**Scope**

For items (i) to (iv) as in relevant sections of Chapters 3 to 6 of the book at Sr.No. 3 of references.

**References:**

1. R.R. Goldberg: Methods of Real Analysis, Oxford and IHB Publishing Company, New Delhi, 1976.
2. S.C.Malik and Savita Arora: Mathematical Analysis, New Age International (P) Ltd, New Delhi, 3<sup>rd</sup> Edition, 2008.
3. H.L.Royden: Real Analysis, Pearson Prentice Hall, Dorling Kindersley (P) Ltd India, 3<sup>rd</sup> Edition 1988.

## Paper-II

### **MATH 723S: Topics in Algebra-II**

**[7 hrs per week (including tutorials)]**

**Max.Marks : 100**

**[Final-80+Internal Assessment-20]**

**Time: 3hrs.**

### **Objective**

*Commutative Algebra is the study of commutative rings, their modules and ideals. This theory has developed over the last 150 years not just as an area of algebra considered for its own sake, but as a tool in the study of two enormously important branches of mathematics: algebraic geometry and algebraic number theory. This course will give the student a background in commutative algebra which is used in both algebraic geometry and number theory.*

- Note :**
- 1. The question paper will have eight questions. Candidates will attempt five questions.**
  - 2. There will be four questions from each part and the students will be required to answer at least two questions from each part.**
  - 3. All questions carry equal marks.**

### **PART-I**

Rings and Modules of fractions, local properties, extended and contracted ideals in ring of fractions, Primary Decomposition, Integral dependence, The going up theorem, Integrally closed domains, The going down theorem, valuations rings.

### **PART-II**

Chain conditions, Noetherian rings, Primary decomposition in Noetherian rings, Noether-Lasker Theorem, Artinian rings.

### **Suggested Books**

1. M.Artin: Algebra, Prentice Hall of India, New Delhi 1994.
2. M.F. Atiyah and I.G. MacDonald: Introduction to Commutative Algebra, Levant Books, Indian Edition, 2007.
3. Nathan Jacobson: Basic Algebra-II, Hindustan Publishing Corporation 1994.
4. R. Y. Sharp: Steps in Commutative Algebra, London Math. Soc. Student Text 19, Cambridge University Press, 1990.

OR

**MATH 724S : Modules and Fields**

[7 hrs per week (including tutorials)]

Max.Marks : 100

[Final-80+Internal Assessment-20]

Time: 3hrs.

**Objective**

*This course is a basic course in Algebra for students who wish to pursue research work in Algebra. Contents have been designed in accordance with the UGC syllabi in mind.*

- Note :**
1. **The question paper will have eight questions. Candidates will attempt five questions.**
  2. **There will be four questions from each part and the students will be required to answer at least two questions from each part.**
  3. **All questions carry equal marks.**

**PART-I**

Modules, Submodules, Quotient Modules, Free Modules, Difference between Modules and Vector Spaces, Homomorphisms, Simple Modules, Structure Theorem of finitely generated modules over a P.I.D., Artinian and Noetherian Modules.

Fields, examples, characteristic of a field. Algebraic extensions, The degree of a field extension, Adjunction of roots, splitting fields, finite fields, Algebraically closed fields,

**PART-II**

Separable and purely inseparable extensions. Perfect fields, primitive elements. Langrange's theorem on primitive elements, normal extensions, Galois extensions, the fundamental theorem of Galois theory. Cyclotomic extensions. Cyclic extensions, Quintic equations and solvability by radicals.

**Suggested Books**

1. David S. Dummit and Richard M Foote: Abstract Algebra, John Wiley & Sons, 2004.
2. I. N. Herstein: Topics in Algebra, 2<sup>nd</sup> Edition, Vikas Publishing House, New Delhi, 1976.
3. C. Musili: Rings and Modules, 2<sup>nd</sup> Revised Edition, Narosa Publishing House, New Delhi, 1994.
4. M. Artin: Algebra, Prentice Hall of India, New Delhi, 1994.
5. P.B. Bhattacharya, S.K. Jain and S.R. Nagpal: Basic Abstract Algebra, 2<sup>nd</sup> Edition, Cambridge University Press, 2002.
6. J. B. Fraleigh: A First Course in Abstract Algebra, 3<sup>rd</sup> Edition, Narosa Publishing House, New Delhi, 3<sup>rd</sup> Edition, 2003.

7. J. A. Gallian: Contemporary Abstract Algebra, 4<sup>th</sup> Edition, Narosa Publishing House, New Delhi, 1998.
8. J.B. Hartley and T. O. Hawkes: Rings, Modules and Linear Algebra, Chapman and Hall.
9. T. W. Hungerford: Algebra, Springer 1974.
10. D.S. Malik, J. N. Mordeson and M. K. Sen: Fundamentals of Abstract Algebra, McGraw-Hill, New York 1997.
11. Surjeet Singh and Q. Zameeruddin, Modern Algebra, 7<sup>th</sup> Edition, Vikas Publishing House, New Delhi, 1993.
12. I.S.Luthar and I.B.S.Passi, Algebra, Vol. 3, Modules, Narosa Publishing House, 2002
13. I.S.Luthar and I.B.S.Passi, Algebra, Vol. 4, Field Thoery, Narosa Publishing House, 2004
14. Mc Carthy, P.J.: *Algebraic Extensions of Fields* (Chelsea), 1976.
15. Nagata, M : *Field Theory*. Matrcel Dekker N.Y. (1977).
16. Winter : *Structure of Fields*. GTM. V. 16 (Springer Verlag), 1974.
17. Stewart-I : *Galois Theory* (Capman & Hall (1973).
18. J.-P. Escofier : *Galois Theory*, Graduate texts in Mathematics, 204, Springer Verlag, 1<sup>st</sup> Edition, 2001.

### Paper-III

#### **MATH 725S : Non-Linear Programming**

[7 hrs per week (including tutorials)]

Max.Marks : 100

[Final-80+Internal Assessment-20]

Time: 3hrs.

#### **Objective**

*To acquaint the students with the concepts of convex and non-convex functions, their properties, various optimality results, techniques to solve nonlinear optimization problems and their duals over convex and non-convex domains and also with the game theory.*

**Note: 1. The question paper will have eight questions. Candidates will attempt five questions.**

**2. There will be four questions from each part and the students will be required to answer at least two questions from each part.**

**3. All questions carry equal marks.**

### **PART-I**

Nonlinear Programming: Convex functions, Concave functions, Definitions and basic properties, subgradients of convex functions, Differentiable convex functions, Minima and Maxima of convex function and concave functions. Generalizations of convex functions and their basic properties.

Unconstrained problems, Necessary and sufficient optimality criteria of first and second order. First order necessary and sufficient Fritz John conditions and Kuhn-Tucker conditions for Constrained programming problems with inequality

constraints, with inequality and equality constraints. Kuhn Tucker conditions and linear programming problems.

## PART -II

Duality in Nonlinear Programming, Weak Duality Theorem, Wolfe's Duality Theorem, Hanson-Huard strict converse duality theorem, Dorn's duality theorem, strict converse duality theorem, Dorn's Converse duality theorem, Unbounded dual theorem, theorem on no primal minimum .Duality in Quadratic Programming.

Quadratic programming: Wolfe's method, Beale's method for Quadratic programming.

Linear fractional programming, method due to Charnes and Cooper. Nonlinear fractional programming, Dinkelbach's approach.

Game theory - Two-person, Zero-sum Games with mixed strategies, graphical solution, solution by Linear Programming.

[Scope as in Chapter 17 of reference no.4, Chapter 3 & 4 of reference no.1, chapter 24, 26 and 28 of reference no.2, Chapter 8 of reference no.3, chapter 16 of reference no.5]

### References:

1. Mokhtar S. Bazaraa & C.M. Shetty: Nonlinear Programming, Theory & Algorithms, 2<sup>nd</sup> Edition, Wiley, New-York, 2004.
2. S.M. Sinha: Mathematical Programming, Theory and Methods, Elsevier, 1<sup>st</sup> Edition, 2006.
3. O. L. Mangasarian: Nonlinear Programming, TATA McGraw Hill Company Ltd. (Bombay, New Delhi), 1<sup>st</sup> Edition, 1969.
4. Kanti Swarup, P.K. Gupta & Man Mohan: Operations Research Sultan Chand & Sons, New Delhi 9<sup>th</sup> Edition, 2001
5. N.S. Kambo, Mathematical Programming Techniques, Affiliated East-West Press Pvt. Ltd. New Delhi, Madras, 1984, Revised Edition, Reprint 2005.

**OR**

**MATH 726S:      Number Theory-II**

**[7 hrs per week (including tutorials)]**

**Max. Marks : 100**

**[Final-80+Internal Assessment-20]**

**Time: 3hrs.**

### **Objective**

*The objectives of this course is to teach the fundamentals of different branches of Number Theory, namely, Geometry of Numbers, Partition Theory and Analytic Number Theory.*

**Note :**    **1. The question paper will consist of two parts containing four questions each. Candidates will attempt five questions.**

2. **There will be four questions from each part and the students will be required to answer at least two questions from each part.**
3. **All questions carry equal marks.**

### **PART-I**

Continued fractions, Approximation of reals by rationals, Pell's equations, Partitions, Ferrers graphs, Jacobi's triple product identity, Congruence properties of  $p(n)$ , Rogers-Ramanujan identities, Minkowski's theorem in geometry of numbers and its applications to Diophantine inequalities.

### **PART-II**

Order of magnitude and average order of arithmetic functions, Euler's summation formula, Abel's identity, Elementary results on distribution of primes. Characters of finite Abelian groups, Dirichlet's theorem on primes in arithmetical progression.

#### **References:**

1. G. H. Hardy and E. M. Wright – An Introduction to Theory of Numbers, Oxford University Press, 6th Ed , 2008.
2. Niven, H. S. Zuckerman and H. L. Montgomery – An Introduction to the Theory of Numbers, John Wiley and Sons, (Asia) 5th Ed., 2004.
3. G. E. Andrews – Number Theory, Dover Books, 1995.
4. T. M. Apostol – Introduction to Analytic Number Theory, Narosa Publishing House, New Delhi (1990)
5. David M. Burton – Elementary Number Theory, Tata McGraw Hill, 6<sup>th</sup> Edition, 2007

### **Paper-IV**

#### **MATH 727S : Complex Analysis-II**

[7 hrs per week (including tutorials)]

Max.Marks : 100

[Final-80+Internal Assessment-20]

Time: 3hrs.

#### ***Objective***

*This course is designed to provide follow up to Course No.707S. This course will provide basic topics needed for students to pursue research in pure Mathematics. Most of the topics mentioned in UGC NET have also been included.*

- Note: 1. The question paper will have eight questions. Candidates will attempt five questions.**
2. **There will be four questions from each part and the students will be required to answer at least two questions from each part.**
  3. **All questions carry equal marks.**

### **PART-I**

Bilinear transformations. Definitions and examples of conformal mappings. Zeros and poles of meromorphic functions, Rouché's theorem, Argument

Principle. Infinite products, Weierstrass theorem, Mittag-Leffler's theorem, Canonical products.

## **PART -II**

Analytic Continuation through power series(basic ideas), Natural boundary, the Gamma function and Riemann Zeta function. Elliptic functions (Scope as in "Complex Analysis" by D. V. Ahlfors Chapter 5 §2.3, 2.4, 4.1, 4.2, Chapter 7§1 to 3.3 and in "Theory of Functions Complex Variable" by Shanti Narayan, Chapter 3, §65-67 of Chapter 5, Chapter 7 §120-129 of Chapter 10).

### **References :**

1. Shanti Narayan: Theory of Functions of a Complex Variable, S. Chand and Co., 7<sup>th</sup> Edition, 1986.
2. D.V. Ahlfors,, Complex Analysis, 3<sup>rd</sup> Edition, 1979 (International student edition), McGraw-Hill International Book Company.
3. J.B. Conway: Function of One Complex Variable, 2<sup>nd</sup> Edition, 1978, Corr 4<sup>th</sup> Print 1986, Graduate Texts, Springer-Verlag, Indian edition by Narosa Publishing House, New Delhi.
4. E. T. Copson: An Introduction to the Theory of Functions of a Complex Variable, The English Language Book Society and Oxford University Press, 1985.
5. K. Knopp: Theory of Functions, (Translated by F. Bagemite) in Two Volumes, Dover Publications, Inc., New York, 1945, 1947.
6. T. Pati: Functions of a Complex Variable, Allahabad, Pothishala, 1971.
7. S. Saks and A. Zygmund: Analytic Functions (Translated by E. J. Scott) Poland, Warszawa, 1952.
8. R. Silverman: Introductory Complex Analysis, Prentice-Hall Inc., Englewood Cliffs, N.J., 1967.
9. J.V. Deshpande: Complex Analysis, Tata McGraw-Hill Publishing Company Ltd., 1989.
10. E.C. Titchmarsh: The Theory of Functions, the English Language Book Society and Oxford University Press, 2<sup>nd</sup> Edition, 1961.
11. Wolfgang Tutschke and Harkrishan L. Vasudeva: An Introduction to Complex Analysis, Classical and Modern Approaches, Chapman and Hall/CRC, 2005.
12. S. Ponnusamy: Foundations of Complex Analysis by Narosa Publishing House, New Delhi, 2<sup>nd</sup> Edition, 2005.

**Paper-V**

**MATH 728S: Classical Mechanics-II**

[7 hrs per week (including tutorials)]

Max.Marks : 100

[Final-80+Internal Assessment-20]

Time: 3hrs.

**Objective**

*This Course is developed to understand the motion of celestial bodies and dynamics of rigid bodies. Introduction to elasticity and elastic waves are incorporated keeping in view their applications in various fields of science and engineering.*

- Note :**
- 1. The question paper will have eight questions. Candidates will attempt five questions.**
  - 2. There will be four questions from each part and the students will be required to answer at least two questions from each part.**
  - 3. All questions carry equal marks.**

**PART-I**

**Central force motion:** Central force, Equivalent one-body problem, Motion in a central force field, General features of the motion: Motion in arbitrary potential field, Motion in a inverse square law, Differential equation of orbit, Classification of orbits, Bertrand's theorem.

**Rigid Body Dynamics:** Moments and product of inertia, Theorems of Parallel and Perpendicular axes, M.I. of thin rod, Rectangular lamina, Rectangular parallelepiped, Circle, Circular disc, Hollow and Solid spheres, Cone etc. Principal axis and examples, Kinetic energy of body rotating about a fixed point, Euler's dynamical equations for motion of rigid body, Solution for symmetrical body, Symmetrical top, Steady motion of Symmetrical top and its Stability, Eulerian angles.

**PART- II**

**Elastodynamics:** Analysis of Deformation tensor, Stresses and condition of equilibrium, Hooke's Law and Strain energy function, Simple cases of strain and stress and equation of motion, Waves in isotropic elastic medium, Waves of dilatation and distortion, Plane waves, Surface waves-Rayleigh and Love waves. Frequency equation of an oscillating sphere, P, SV and SH waves, Reflection of P and SH waves from stress free boundary surface of a uniform elastic half-space.

**References:**

1. H. Goldstein, C. Poole and J. Safko: Classical Mechanics, 3<sup>rd</sup> Ed., Addison Wesley (2002)
2. F. Chorlton: Text book of Dynamics, CBS Publishers (1985)
3. F. Grantmacher: Lecture in analytical Mechanics, Mir Publication, 1975.
4. I.S. Sokolnikoff: Mathematical Theory of Elasticity, Tata McGraw Hill Publishing Co., New Delhi(1977)
5. Kolsky, H: Stress Waves in Solids, Dover Publication, Inc (1963).
6. Ghosh, P K: Mathematics of Waves and Vibrations, The Macmillan Company of India Limited (1975)



**OUTLINES OF TESTS, SYLLABI AND COURSES OF READING FOR M.SC. (HONS. SCHOOL) IN MATHEMATICS SEMESTER III & IV FOR THE ACADEMIC SESSION 2015-2016.**

**Outlines of Tests**

Every student will have to take **five papers** from the following list depending upon his/her background. Students who have studied Math-702S & 722S, Math-704S & 724S and Math-706S & 726S in M.Sc.(Hons. School) Sem.I & II will have to take Math-705S & 725S, Math-751S & 770S and Math-752S & 769S as compulsory papers and any two from Math-761S & 781S, Math-771S & 791S, Math-772S & 792S, Math-773S & 793S, Math-774S & 794S, Math-776S & 797S, Math-777S & 797S and Math 778S & 798S (except Paper:Math-775S & 795S)in Sem.III & IV. Students who have studied Math-701S & 721S, Math-703S & 723S and Math-705S & 725S in M.Sc.(Hons. School) Sem. I & II will have to choose any five papers from Math-771S & 791S, Math-772S & 792S, Math-773S & 793S, Math-774S & 794S, Math-775S & 795S, Math-776S & 796S, Math-777S & 797S and Math 778S & 798S in Sem.III & IV.

**Semester III**

1. Math 705S: Linear Programming
2. Math 751S: Topology
3. Math 752S: Linear Algebra and Commutative Algebra-I
4. Math 761S: Computational Techniques-I
5. Math 771S: Algebraic Number Theory-I
6. Math 772S: Topics in Number Theory-I
7. Math 773S: Fluid Mechanics-I
8. Math 774S: Algebraic Coding Theory-I
9. Math 775S: Non-Commutative Ring Theory
10. Math 776S: Partial Differential Equations-I
11. Math 777S: Continuum Mechanics-I
12. Math 778S: Numerical Methods for Differential Equations-1
13. Math-779S: Network Analysis
14. Math-780S: Measure & Integration-I

**Semester IV**

1. Math 725S: Non-Linear Programming
2. Math 770S: Functional Analysis
3. Math 769S: Commutative Algebra-II
4. Math 781S: Computational Techniques-II
5. Math 791S: Algebraic Number Theory-II
6. Math 792S: Topics in Number Theory-II
7. Math 793S: Fluid Mechanics-II
8. Math 794S: Algebraic Coding Theory-II
9. Math 795S: Representation Theory of Finite Groups
10. Math 796S: Partial Differential Equations-II
11. Math 797S: Continuum Mechanics-II]
12. Math 798S: Numerical Methods for Differential Equations-1
13. Math-799S: Numerical Optimization
14. Math-800S:Measure and Integration-II

## Semester-III

### MATH 705S : Linear Programming

[7 hrs per week (including tutorials)]

Max.Marks : 100

[Final-80+Internal Assessment-20]

Time: 3hrs.

#### Objective

*The objective of this course is to acquaint the students with the concept of convex sets, their properties and various separation theorems so as to tackle with problems of optimization of functions of several variables over polyhedron and their duals. The results, methods and techniques contained in this paper are very well suited to the realistic problems in almost every area.*

- Note:**
- 1. The question paper will have eight questions. Candidates will attempt five questions.**
  - 2. There will be four questions from each part and the students will be required to answer at least two questions from each part.**
  - 3. All questions carry equal marks.**

#### PART-I

Linear Programming and examples, Convex Sets, Hyperplane, Open and Closed half-spaces, Feasible, Basic Feasible and Optimal Solutions, Extreme Point & graphical methods. Simplex method, Charnes-M method, Two phase method, Determination of Optimal solutions, unrestricted variables, Duality theory, Dual linear Programming Problems, fundamental properties of dual Problems, Complementary slackness, Unbounded solution in Primal. Dual Simplex Algorithm, Sensitivity analysis.

#### PART-II

Parametric Programming, Revised Simplex method, Transportation Problems, Balanced and unbalanced Transportation problems, U-V method, Paradox in Transportation problem, Assignment problems, Integer Programming problems: Pure and Mixed Integer Programming problems, 0-1 programming problem, Gomory's algorithm, Branch & Bound Technique. Travelling salesman problem [scope as in reference no 2].

[Scope as in Chapter 2-5; Chapter 7-9 of the reference no.1, chapter 4-6 of reference no3, chapter 5 of reference no2].

#### References:

1. G.Hadley: Linear Programming, Narosa Publishing House, 6<sup>th</sup> Edition. 1995.
2. N.S. Kambo: Mathematical Programming Techniques, 1984, Affiliated East-West Press Pvt.Ltd. New Delhi, Madras(Reprint 2005) revised Edition.
3. Suresh Chandra, Jayadeva and Aparna Mehra: Numerical Optimization with Applications, Narosa Publishing House, 1<sup>st</sup> Edition, 2009.
4. S.M.Sinha, Mathematical Programming, Theory and Methods, 1<sup>st</sup> Edition, Elsevier, 2006.

**MATH-751S: Topology**

[7 hrs per week (including tutorials)]

Max.Marks : 100

[Final-80+Internal Assessment-20]

Time: 3hrs.

**Objectives**

The course is an introductory course on point-set topology so as to enable the reader to understand further deeper topics in topology like Differential/Algebraic Topologies etc.

- Note :**
- 1. The question paper will have eight questions. Candidates will attempt five questions.**
  - 2. There will be four questions from each part and the students will be required to answer atleast two questions from each part.**
  - 3. All questions carry equal marks.**

**PART – I**

Topological Spaces, bases for a topology, the order topology, the product topology on  $X \times Y$ , the subspace topology, closed sets and limit points, continuous functions, the product topology, the metric topology, the quotient topology, Sequence, Nets and Filters

Connected spaces, connected subspaces of the real line, components and local connectedness

[Scope as in the relevant sections in Chapter 2 & 3 of the book 'Topology', second edition 2002, by James R. Munkres.]

**PART –II**

Compact spaces, compact space of the real line, limit point compactness, local compactness, nets.

The countability axioms, the separation axioms, normal spaces, the Urysohn Lemma, the Urysohn Metrization Theorem, the Tietze Extension Theorem, the Tychonoff Theorem.

[Scope as in the relevant sections in Chapters 3, 4 and 5 of the book 'Topology', second edition 2002, by James R. Munkres.]

**References:**

1. James R. Munkers : Topology, 2<sup>nd</sup> Edition, 2002, Prentice Hall of India.
2. James Dugundji : Topology, UBS Publishers, 1<sup>st</sup> Edition, 1990. .
3. John L. Kelley : General Topology (Van Nostrand), 1<sup>st</sup> Edition, 1955.
4. Bourbaki – General Topology (Reading, Addison-Wesley), 1<sup>st</sup> Edition, 1989.
5. G.G. Simmons - Introduction to Topology and Modern Analysis Tokyo, McGraw Hill, Kongakusha), 1<sup>st</sup> Edition, 1963.
6. W.J. Thron, - Topological structures (N.Y.Holt) (Scope as in Chapters IV to XV, Chapter XVI: def. 16.4 and results including Tychonoff's theorem and Chapter XVIII of the reference 4), 1<sup>st</sup> Edition, 1966..
7. E.T. Copson – Metric Spaces (Cambridge University Press), 1<sup>st</sup> Edition, 1963.
8. S. Willord – General Topology (Addison Wesley Publishing Company), 1970.

**Math 752S: Linear Algebra and Commutative Algebra I**

[7 hrs per week (including tutorials)]

Max.Marks : 100

[Final-80+Internal Assessment-20]

Time: 3hrs.

**Objective**

*The objectives of this course is to develop a strong foundation in linear Algebra that provide a basis for advanced studies not only in Mathematics but also in other branches like engineering, physics and computers etc. Particular attention to canonical forms of linear maps, matrices, bilinear forms and quadratic forms is given. The aim is to develop necessary prerequisites for Math 769S.*

- Note :**
- 1. The question paper will have eight questions. Candidates will attempt five questions.**
  - 2. There will be four questions from each part and the students will be required to answer atleast two questions from each part.**
  - 3. All questions carry equal marks.**

**PART-I**

Linear Algebra: Eigenvalues and eigenvectors, eigenspaces and similarity, representation by a diagonal matrix; linear functionals, real quadratic forms, orthogonal matrices, reduction of real quadratic forms, classification of real quadratic forms, bilinear forms, symmetric bilinear forms, hermitian forms; inner product spaces, norms and distances, orthonormal bases, orthogonal complements, isometries, normal matrices, normal linear operators.

**PART-II**

Projections and direct sums, spectral decompositions, minimal polynomials and spectral decompositions, nilpotent transformations, the Jordan canonical form.

Commutative Algebra: Rings and ideals, modules, tensor products of modules.

**References**

1. S.H. Friedberg, A.J. Insel, L.E. Spence: Linear Algebra, Prentice Hall, 2003.
2. J. Gilbert and L. Gilbert: Linear Algebra and Matrix Theory, Academic Press, 1995.
3. I.G. MacDonald and M.F. Atiyah: Introduction to Commutative Algebra, Addison-Wesley 1969; reprinted by Perseus 2000.
4. Gopalakrishnan, N.S., Commutatilve Algebra, Oxonian Press (New Delhi) 1984
5. O. Zariski and P. Samuel, Commutative Algebra, vols I and II, Springer 1975.

## MATH 761S: Computational Techniques –I

### Theory

[4 hrs per week (including tutorials)]

Max. Marks: 80

[Final-60+Internal Assessment-20]

Time: 3hrs.

### **Objective**

*The objective of this course is to teach the basics of computer and computer programming so that one can develop the computer program in FORTRAN at their own. For the purpose of learning programming skill, some Numerical methods which are extremely useful in scientific research are included. For practising the programmes of the numerical method, the course of practical has also been included in this paper. The contents of the curriculum have been designed keeping in view the UGC guidelines.*

- Note :**
- 1. The question paper will have eight questions. Candidates will attempt five questions.**
  - 2. There will be four questions from each part and the students will be required to answer at least two questions from each part.**
  - 3. All questions carry equal marks.**
  - 4. Use of scientific calculator is allowed for numerical work.**

### **PART – I**

**FORTRAN77:** Character set, constants, variables, Arithmetic expressions, Library functions, Arithmetic statements, Structure of a FORTRAN Program, FORMAT specifications, READ and WRITE statements, Simple programs, Control statements: GO TO, IF, IF-THEN-ELSE and ELSE-IF-THEN statements, DO loop, Nested DO loop, CONTINUE statement, DATA statement, DOUBLE precision, LOGICAL data, WHILE structure, Arrays and Subscripted variables, Implied DO loop, One and multi-dimensional arrays, Sub programs: Function subprogram and Subroutine subprogram, OPEN a file, Read from a file , Write in a file.

### **PART- II**

**Solution of non-linear equations:** Bisection, Regula-falsi, Secant, Newton-Raphson, Generalized Newton's method, Chebyshev Formula of third order, Halley's methods, Functional iteration, Muller's methods, Convergence analysis of these methods, Comparison of these methods, Simultaneous non-linear equations by Newton-Raphson method, Lin- Bairstow's and Newton's method for complex roots.

**Interpolation:** Finite differences, Newton's formulae for interpolation, Lagrange and Hermite interpolation, Cubic Spline interpolation.

**Numerical integration**-Trapezoidal, Simpson's, Boole's, Weddle's rule, Error in Integration formulae, Double Integration, Truncation errors in Trapezoidal and Simpson's rules.

### **Computational Techniques (Practical)-I**

**[3 hrs per week, Max. Marks: 20]\***

Writing programs in FORTRAN for the problems based on the method studied in theory paper and run them on PC.

Practical shall be conducted by the department as per the following distribution of marks:

Writing program in FORTRAN and running it on PC=10 Marks

Practical record=5 Marks

Viva-Voice=5 Marks

#### **References:**

1. C. Xavier: FORTRAN 77 and Numerical Methods, New Age Int. Ltd., 1994.
2. S. S. Shastry: Introductory Methods of Numerical Analysis, PHI, 2005.
3. C. F. Gerald and P. O. Wheatley: Applied Numerical Analysis, Pearson Education, Asia, 2004.

**\*NOTE: There will be no internal assessment in the Practical examination.**

### **MATH: 771S: Algebraic Number Theory-I**

**[7 hrs per week (including tutorials)]**

**Max.Marks : 100**

**[Final-80+Internal Assessment-20]**

**Time: 3hrs.**

#### **Objective**

*This course is an introduction to algebraic number theory which is a subject that originated as a result of the attempts to solve Fermat's Last Theorem. Algebraic number theory is an active area of research in mathematics.*

- Note :**
- 1. The question paper will have eight questions. Candidates will attempt five questions.**
  - 2. There will be four questions from each part and the students will be required to answer atleast two questions from each part.**
  - 3. All questions carry equal marks.**

**PART-I**

Algebraic number fields and their rings of integers, Integral bases, Discriminant, Explicit consideration of quadratic, cyclotomic and special cubic fields. Properties of norm of ideals in the ring of algebraic integers, Factorization of ideals into prime ideals.

**PART -II**

Dirichlet's Theorem on units, Dedekind's theorem for decomposition of rational prime in algebraic number fields, splitting of rational primes in cyclotomic fields.

**References**

1. D.A.Marcus: Number Fields, Springer-Verlag, New York, 1977.
2. R.A.Mollin: Algebraic Number theory, Chapman & Hall/CRC, 2011.
3. P. Samuel: Algebraic Theory of Numbers, Dover Publications, 1970.
4. P. Ribenboim: Classical Theory of Algebraic Number, Springer-Verlag, New York, 2001.
5. I. Stewart and D. Tall: Algebraic Number theory, 2<sup>nd</sup> Edition, Chapman & Hall, 1907.

**MATH: 772S Topics in Number Theory –I**

[7 hrs per week (including tutorials)]

Max.Marks : 100

[Final-80+Internal Assessment-20]

Time: 3hrs.

**Objective**

*The objective of this course is to familiarize the students with the theory of basic series and their applications in many Number Theory problems, particularly in Partition Theory.*

- Note :**
- 1. The question paper will have eight questions. Candidates will attempt five questions.**
  - 2. There will be four questions from each part and the students will be required to answer atleast two questions from each part.**
  - 3. All questions carry equal marks.**

**PART-I**

Basic hypergeometric series, q-binomial theorem, Heine's transformation, q-Gauss theorem, Restricted partitions, Gaussian polynomials, q-Saalschutz's theorem, Bailey's lemma (weak version), Rogers lemma, Rogers-Ramanujan identities, Schur's theorem, Gordon-Gollnitz identities, Generalization and various analogues of Rogers-Ramanujan identities.

## PART-II

Bailey's lemma (strong version), Watson's q-analogue of Whipple's theorem and its applications in deriving Rogers-Ramanujan identities and Gordon-Gollnitz identities.  $6\text{-}\phi\text{-5}$  identity and its applications to representations of numbers as sum of two squares, four squares and four triangular numbers. Frobenius partitions, coloured Frobenius partitions. Plane partitions.

### Suggested Readings

1. A.K. Agarwal, Padmavathamma and M.V. Subbarao, Partition Theory, Atma Ram & Sons, Chandigarh, 2005.
2. G.E. Andrews, The Theory of Partitions, Encyclopedia of Mathematics and its Applications (Addison-Wesley), 1976, Re-issued: Cambridge University Press, Cambridge, 1988.
3. G.Gasper and M. Rahman, Basic Hypergeometric Series, Encyclopedia of Mathematics and its Applications, Vol. 35 Cambridge University Press, Cambridge, 1990.
4. R.P. Agarwal, Resonance of Ramanujan Mathematics, Vol.1 (New Age International), 1996.
5. H. Gupta, Selected Topics in Number Theory, ABACUS Press, 1980.
6. G.E. Andrews, Generalized Frobenius Partitions, Memoirs of the American Mathematical Society, Vol.49, No.301, 1984.

### MATH-773S: Fluid Mechanics-I

[7 hrs per week (including tutorials)]

Max.Marks : 100

[Final-80+Internal Assessment-20]

Time: 3hrs.

#### Objective

*The objective of this course is to introduce to the fundamentals of the study of fluid motion and to the analytical approach to the study of fluid mechanics problems.*

- Note :**
- 1. The question paper will have eight questions. Candidates will attempt five questions.**
  - 2. There will be four questions from each part and the students will be required to answer atleast two questions from each part.**
  - 3. All questions carry equal marks.**

## PART-I

Real fluids and ideal fluids, velocity of fluid at a point, streamlines, pathlines, streaklines, velocity potential, vorticity vector, local and particle rate of change, equation of continuity, irrotational and rotational motion, acceleration of fluid, conditions at rigid boundary.



Euler's equation of motion, Bernoulli's equation, their applications, Potential theorems, axially symmetric flows, impulsive motion, Kelvin's Theorem of circulation, equation of vorticity.

### **PART-II**

Some three dimensional flows: sources, sinks and doublets, images in rigid planes, images in solid sphere, Stoke's stream function.

Two dimensional flows: complex velocity potential, Milne Thomson Circle Theorem and applications, Theorem of Blasius, vortex rows, Karman vortex street.

### **References**

1. Chorlton, F.: Text Book of Fluid Dynamics, CBS Publishers, Indian Edition, 2004.
2. L.D.Landau & E. N. Lipschitz: Fluid Mechanics, 2<sup>nd</sup> Edition, Vol. 6 (Course of Theoretical Physics), Pergamon Press Ltd., 1987.
3. G. K. **Batchelor**: An Introduction to Fluid Mechanics, Cambridge University Press, 1967.
4. Kundu and Cohen: Fluid Mechanics, Indian Reprint, Published by Harcourt (India) Pvt.Ltd., 2003.

### **Math 774S: Algebraic Coding Theory-I**

[7 hrs per week (including tutorials)]

Max.Marks : 100

[Final-80+Internal Assessment-20]

Time: 3hrs.

### **Objective**

*Coding theory is concerned with successfully transmitting data through a noisy channel and correcting errors in corrupted messages. The objectives is to introduce a first course of coding theory, the algebraic structure of linear codes, some special codes and several bounds in coding theory.*

**Note : 1.The question paper will have eight questions. Candidates will attempt five questions.**

**2. There will be four questions from each part and the students will be required to answer atleast two questions from each part.**

**3. All questions carry equal marks.**

### **PART-I**

Error detecting and error correcting codes, maximum likelihood decoding, Hamming distance, Finite Fields, Linear Codes, Generator matrix and parity check matrix, Dual Codes, Syndrome Decoding, Weight Enumerator of a Code, Macwilliam equations, Macwilliam's Identity, ISBN Codes, New Codes from old.

## PART-II

Sphere covering bound, Sphere packing bound, Gilbert Varshamov bound, perfect codes, Hamming Codes, Golay codes, Simplex Codes, Singleton bound and MDS codes, Plotkin bound, Griesmer bound, Reed-Muller codes, Linear Programming bounds. The Johnson Upper bounds.

### References

1. San Ling and Chaoping Xing- Coding Theory, Cambridge University Press, 1<sup>st</sup> Edition, 2004.
2. W. C. Huffman and Vera Pless – Fundamentals of Error Correcting Codes, Cambridge University Press, 1<sup>st</sup> South Asian Edition, 2004
3. Raymond Hill- Introduction to Error Correcting Codes, Oxford University Press, 1986, reprint 2009.
4. F. J. MacWilliams and N.J.A.Sloane – Theory of Error Correcting Codes Part I & II, Elsevier/North-Holland, Amsterdam, 1977
5. Vera Pless - Introduction to Theory of Error Correcting Codes, Wiley-Interscience, 3<sup>rd</sup> Edition, 1982.

## Math 775S-Non Commutative Ring Theory

[7 hrs per week (including tutorials)]

Max.Marks : 100

[Final-80+Internal Assessment-20]

Time: 3hrs.

### Objective

*This course is a prerequisite for the course Math 795S on Representation theory of finite groups, which is a subject of great importance.*

- Note :**
- 1. The question paper will have eight questions. Candidates will attempt five questions.**
  - 2. There will be four questions from each part and the students will be required to answer atleast two questions from each part.**
  - 3. All questions carry equal marks.**

### PART-I

Non-commutative rings and left/right Modules over them, Modules of finite length, Artinian and Noetherian Modules, Artinian and Noetherian Rings, Triangular Rings, Semi-simple Modules, Isotypical Components, Endomorphism Rings, Semi-simple Rings and Wedderburn- Artin's Theorem.

### PART-II

The Jacobson Radical, Radical of an Artinian Ring, J-semisimplicity of Rings, Tensor Product of Modules over Non-Commutative rings, Tensor Product of Algebras, Central Simple Algebras, Skolem-Noether's Theorem, Double-Centralizer theorem, Brauer Groups, Brauer Groups of  $R$ , Relative Brauer Groups and splitting fields of Central Simple Algebras.

The Group Algebras and their augmentation ideals.

**Suggested Books**

1. I. N. Herstein, Non-Commutative Rings, The Carus Mathematical Monograph, The Mathematical Association of America, 1968.
2. C. Musili, Representations of Finite Groups, Hindustan Book Agency, 1993.
3. J. -P. Serre, Linear Representations of Finite Groups, Graduate Texts in Mathematics; 42, Springer Verlag, 1977.
4. T. Y. Lam, A First Course in Non-commutative Rings, Graduate Texts in Mathematics; 131, Springer Verlag, 1991.

**MATH 776S: Partial Differential Equations –I**

[7 hrs per week (including tutorials)]

**Max.Marks : 100****[Final-80+Internal Assessment-20]****Time: 3hrs.****Objective**

*The objective of this course is to equip the students with knowledge of some advanced concepts related to partial differential equations and to understand some basic approach to mathematical oriented PDEs.*

- Note :**
- 1. The question paper will have eight questions. Candidates will attempt five questions.**
  - 2. There will be four questions from each part and the students will be required to answer atleast two questions from each part.**
  - 3. All questions carry equal marks.**

**PART– I**

Partial Differential equations of 2<sup>nd</sup> and Higher order, Classification, Examples of PDE, Solutions of Elliptic, Hyperbolic and Parabolic equations. Transport equation-Initial value problem, Non-homogeneous equations, Laplace's equation-Fundamental solution, Mean value Formulas, Properties of harmonic functions, Green functions, Energy Methods, Heat equation- Fundamental solution, Mean value formulas, Properties of solutions,

**PART– II**

Wave equation- Solution by spherical means, non-homogeneous problem, Energy methods. Non-Linear first order PDE: Complete Integrals, Envelopes, Characteristics, Hamilton-Jacobi equations, Hamilton's ODE, Legendre transform, Hopf – Lax formula,

**References:**

1. L. C. Evans: Partial Differential equations, Graduate Studies in Mathematics Vol 19, American Mathematical Society, (1998).
2. Robert C. McOwen: Partial Differential Equations methods and applications, 2<sup>nd</sup> Edition, Pearson Education Inc., 2003.
3. H.F.Weinbergergerger: A first course in Partial Differential Equations with complex variables and transform methods. Corrected reprint of the 1965 original, dover Publications, Inc., NY, 1995.

4. P. Prasad and R.Ravindran.Partial differential equations. New Age international. 2012

### **MATH 777S: Continuum Mechanics-I**

[7 hrs per week (including tutorials)]

Max.Marks : 100

[Final-80+Internal Assessment-20]

Time: 3hrs.

#### **Objective**

This course introduces the tensors and mathematical aspects of the basic concepts of elastic body deformation. This is very helpful in understanding the mechanics of elastic bodies.

- Note :**
1. **The question paper will have eight questions. Candidates will attempt five questions.**
  2. **There will be four questions from each part and the students will be required to answer atleast two questions from each part.**
  3. **All questions carry equal marks.**

#### **PART – I**

**Tensors:** Summation convention, coordinate transformation, Cartesian tensor of various orders, algebra of tensors, contraction, symmetric and skew-symmetric tensor. Kronecker delta, Alternating tensor, Gradient, Divergence, Curl in tensor notations, Gauss-divergence theorem, partial derivatives, contravariant and covariant tensors, metric tensor, physical components.

**Strain Analysis:** Affine transformation, infinitesimal affine transformation, geometrical interpretation of components of strain, strain quadric of Cauchy, strain-displacement relation.

#### **PART – II**

**Strain Analysis** (continued): Strain invariants, compatibility, principal direction and principal strain, homogeneous deformation.

**Stress Analysis:** Stress vector and stress tensor, symmetry of stress tensor, stress quadric of Cauchy, equation of equilibrium and motion, principal direction of stress, Mohr's diagram.

#### **References:**

1. Shanti Narayan: Cartesian Tensor, 3<sup>rd</sup> Edition, 1968.
2. Young, E. C.: Vectors and Tensor Analysis, 2<sup>nd</sup> Edition, 1993.
3. Sokolnikoff, I. S.: Mathematical theory of elasticity, Mc-Graw-Hill, 2<sup>nd</sup> Edition, 1982.

## **MATH 778S : Numerical Methods for Differential Equations-I**

### **Theory**

**[4 hrs per week theory (including tutorials)]**

**Max. Marks: 80**

**[Final-60+Internal Assessment-20]**

**Time: 3hrs.**

### **Objective**

*The aim of this course is to teach the basics of MATLAB package. At the end of the course, the students will be able to do programming in MATLAB and understand the basic concepts in Numerical Analysis of differential equations.*

### **Note:**

- 1. The question paper will have eight questions. Candidates will attempt five questions.**
- 2. There will be four questions from each part and the students will be required to answer at least two questions from each unit.**
- 3. All questions carry equal marks.**

### **PART-I**

Basics of MATLAB: MATLAB as a calculator, Defining Variables, Display format, Saving the variables stored in memory, Predefined variable, Complex numbers, Vectors and Matrices.

Control Flow: If-end, If-else-end, Elseif, Switch-case, For loops: Single for loops, Nested for loops, Special cases of the for loop, While loops.

Functions: General Structure of function, Scope of variables, Passing variable, The Return statement, Nargin and nargout, Recursive functions.

Plotting: Basic two-dimensional plots, Line styles, Markers, Colors, Plot Color, Plotting grid, Axis command, Placing text on a plot, Modifying text with Tex commands.

Polynomial splines and Generalizations: Cubic splines, Definition of cubic and m splines, Derivation of B splines, Quintic spline interpolate, Splines and ordinary differential equations, Error analysis.

The method of collocation : Introduction, A simple special case, existence via matrix analysis, Green's functions, Collocation existence via green's functions, Error analysis via Green's function, Collocation and partial differential equations, Orthogonal collocation, A connection between Collocation and Galerkin methods.

### **PART-II**

Basic steps of finite element analysis: Model boundary value problem, Descretization of the domain, Derivation of element equation, Connectivity of

elements, Imposition of boundary conditions, Solution of equations, Post processing of the solution, Radially symmetric problems.

Finite Element Error Analysis: Approximation Errors, Various Measure of Errors, Convergence of solutions, Accuracy of the solution.

Finite Element Analysis of two Dimensional Problems : The model equation, Finite element discretization, Weak form of the problem, Finite element formulation, Interpolation functions, Linear triangular element, linear rectangular element, Evaluation of element matrices and vectors, assembly of element equations, post processing, Axisymmetric problems, Finite element formulation for Poisson' problem.

## Practical

[3 hrs per week, Max. Marks: 20]

Time: 3hrs

Writing programs in Matlab for the following problems and run them on PC.

1. Write a program in Matlab to solve a polynomial equation.
2. Write a program in Matlab to find  ${}^n C_r$ .
3. Write a program in Matlab to write a tridiagonal matrix.
4. Write a program in Matlab to solve the system of linear equations
  - a) using Gauss Elimination
  - b) using LU Decomposition.
5. Write a program in Matlab to find the characteristic roots and the characteristic functions
6. Let  $\mathbf{U} = [U_1, U_2, \dots, U_N]$  be a numerical solution of a problem and  $\mathbf{u} = [u(x_1), u(x_2), \dots, u(x_N)]$  be the exact solution of the problem at grid points. Write a program in Matlab to find the absolute error
  - a) in maximum norm or infinity norm.
  - b) in  $L_2$  norm.
7. Write a Program in Matlab to find the numerical solution using Euler's method (forward and backward numerical scheme) and compare it with the exact solutions.

$$y' = t/y \quad ; \quad 0 \leq t \leq 5$$

$$y(0) = 1$$

8. Write a program in Matalab to solve the following problem

$$h' + 0.002(52.1h + (10.3/(10.3+h))) - 1.17(1 + \sin 3t) = 0.0308, \quad h(0) = 5.0.$$

9. A simple model for the falling body with the initial conditions is

$$y'' = -1 + y'^2$$

$$y(0) = 1, \quad y'(0) = 0.$$

Write a program in Matlab to find the value of  $t$  for which  $y(t) = 0$ ?

10. A model of flame propagation, when we light a match, the ball of flame grows rapidly until it reaches a critical size. Then it remains at that size because the amount of oxygen being consumed by the combustion in the interior of the ball balance the amount available through.

$$y' = y^2 - y^3; \quad y(0) = p, \quad 0 \leq t \leq 2/p$$

Here,  $y(t)$  represents the radius of the ball. The  $y^2$  and  $y^3$  terms come from the surface area and volume. The critical parameter is the initial radius  $p$ . Write a program in Matlab to find the numerical solution.

11. Consider the initial value

$$x' = -(1 + t + t^2) - (2t - 1)x - x^2, \quad 0 \leq t \leq 3, \quad x(0) = -1/2.$$

The exact solution is given by

$$x(t) = -t - 1/(e^t + 1).$$

Write a Program in Matlab to find the numerical solution and compare it with the exact solution.

12. A mass-spring system can be modeled via the following second-order ODE

$$Y'' + cy' + w^2y = g(t), \quad y(0) = 1, \quad y'(0) = 0.$$

Write a program in Matlab to find the numerical solution for the particular set of conditions  $c = 5$ ,  $w = 2$  and  $g(t) = \sin(t)$

13. Write a program in Matlab to evaluate shape functions for  
 a) Three node element.  
 b) Four node element.
14. Write a program in Matlab to find the numerical solution the reaction-diffusion problem defined on  $(0,1)$  with the homogeneous boundary conditions using method of collocation.
15. Write a program in Matlab to find the numerical solution the convection-diffusion problem defined on  $(0, 1)$  with the homogeneous boundary conditions using method of collocation.
16. Write a program in Matlab to find the numerical solution of the following problem

$$y'' = -2, \quad 0 < x < 1$$

$$y(0) = 0, \quad y'(0) = 0$$

using finite element method.

17. Write a program in Matlab to find the numerical solution of the Poisson equation defined on a square region with Dirichlet boundary condition using finite element method.

Practical shall be conducted by the department as per the following distribution of marks.

Writing program in Matlab and running it on PC=10 Marks

Practical record=5 Marks

Viva-Voice=5 Marks

### References:

1. P M Prenter, Splines and Variational Methods, illustrated, Reprint Edition, Dover Publications, 2008.
2. J. N. Reddy, An Introduction to the Finite Element Method, II<sup>nd</sup> Edition, TATA McGraw-Hill, 2003
3. Erick G. Thompson, Introduction to the Finite Element Method, John Wiley & Sons, Inc., 2005.
4. Gilbert Strang and George J. Fix, An Analysis of the Finite Element Method, II<sup>nd</sup> Edition, Wellesley-Cambridge Press, 2008.
5. Marc E. Herniter, Programming in Matlab, CL-Engineering, 2000.
6. Desmond J. Higham and Nicholas J. Higham, MATLAB Guide, Second Edition, SIAM 2005.
7. C. B. Moler, Numerical Computing with MATLAB, SIAM, Philadelphia, 2004.

### Math 779S: Network Analysis

**[7 hrs per week (including tutorials)]**

**Max.Marks : 100**

**[Final-80+Internal Assessment-20]**

**Time: 3hrs.**

### Objective

To acquaint students with theoretical development of various algorithms for network flow problems and their application in solving maximal flow, minimal cost, network flow problems and shortest path problems.

- Note :**
- 1. The question paper will have eight questions. Candidates will attempt five questions.**
  - 2. There will be four questions from each part and the students will be required to answer atleast two questions from each part.**
  - 3. All questions carry equal marks.**



## PART I

Flows in Networks: Minimal Cost Network Flow problem, some basic definitions and terminology from Graph Theory. Properties of the A matrix , representation of a non basic vector in terms of the basic vectors.The Simplex method for Network flow problem and examples.Finding an initial basic feasible solution.

Network flows with lower and upper bounds,The Simplex table associated with a network flow problem.

The Out-of Kilter formulation of Minimal cost Network flow problem. Strategy and theoretical development of the Out-of Kilter algorithm.A labeling procedure for the Out-of Kilter algorithm, insight into the changes in primal and dual function values.

Scope as in chapter 9 (9.1-9.9) and chapter 11 (11.1-11.6) of Ref. No.1.

## PART II

The Maximal flow problem: Cut set and its capacity, Dual of max flow problem, Maximal flow-minimal cut theorem, an algorithm for Maximal flow problem, a labelling technique for Max flow problem.

Shortest path problem: Dijkstra's algorithm for shortest path problem for all non negative costs.Theoretical development, validity and complexity of the algorithm, a primal simplex interpretation of the algorithm. A shortest path procedure for arbitrary costs, a labelling algorithm for shortest path problem.

Multi commodity minimal cost flow problem. Scope as in chapter 12(12.1,12.2,12.4,12.5) of Ref. No.1.

### **References**

1. Mokhtar S. Bazaraa, John J. Jarvis and Hanif D. Sherali, Linear programming and Network Flows, John Wiley and Sons, 2004.
2. Ford, L.R. and Fulkerson, D. R, Flows in Networks, Princeton University Press,2010.
3. Ahuja, R. K., Magnati, T. L. Network Flows-Theory, Algorithm and Applications, Prentice Hall, N.J., 2005.

**Math 780S: Measure and Integration-I****[7 hrs per week (including tutorials)]****Max.Marks : 100****[Final-80+Internal Assessment-20]****Time: 3hrs.****Objective**

The objective of this course is to study measure in an abstract setting after having studied Lebesgue measure on real line. The general  $L^p$  spaces and complex measure are also studied.

**Note :** 1. **The question paper will have eight questions. Candidates will attempt five questions.**

2. **There will be four questions from each part and the students will be required to answer atleast two questions from each part.**

3. **All questions carry equal marks.**

**PART I**

Abstract integration, the concept of measurability, simple functions, elementary properties of measures, integration of positive functions, integration of complex functions, the role played by sets of measure zero.

Positive Borel measures: vector spaces, topological preliminaries, The Riesz representation Theorem. Regularity properties of Borel measures, Lebesgue measure, continuity properties of measurable functions.

**PART II**

$L^p$ -spaces: Convex functions and inequalities, The  $L^p$ - spaces, Approximations by continuous functions.

Review of trigonometric series, Fourier coefficients of  $L^1$ -functions.

Complex measures: Total variation, absolute continuity, Radon- Nikodym theorem, Bounded linear functional on  $L^p$ - spaces, the Riesz representation theorem.

(Scope as in Chapters 1,2,3,4,5 & 6 of the book at Ref.No. 3)

**References:**

1. E. Hewitt and K. A. Ross: Abstract Harmonic Analysis, Springer Verlag , Berlin, Vol. I , 1963 , Vol. II, 1970.
2. H. L. Royden: Real Analysis, third edition , Pearson Prentice Hall, 2009.
3. W. Rudin: Real and Complex Analysis, third edition TMH, New Delhi, 2010.

## Paper-IV

### **MATH 725S : Non-Linear Programming**

[7 hrs per week (including tutorials)]

Max.Marks : 100

[Final-80+Internal Assessment-20]

Time: 3hrs.

#### **Objective**

*To acquaint the students with the concepts of convex and non-convex functions, their properties, various optimality results, techniques to solve nonlinear optimization problems and their duals over convex and non-convex domains and also with the game theory.*

- Note:**
- 1. The question paper will have eight questions. Candidates will attempt five questions.**
  - 2. There will be four questions from each part and the students will be required to answer at least two questions from each part.**
  - 3. All questions carry equal marks.**

#### **PART-I**

Nonlinear Programming: Convex functions, Concave functions, Definitions and basic properties, subgradients of convex functions, Differentiable convex functions, Minima and Maxima of convex function and concave functions. Generalizations of convex functions and their basic properties.

Unconstrained problems, Necessary and sufficient optimality criteria of first and second order. First order necessary and sufficient Fritz John conditions and Kuhn-Tucker conditions for Constrained programming problems with inequality constraints, with inequality and equality constraints. Kuhn Tucker conditions and linear programming problems.

#### **PART -II**

Duality in Nonlinear Programming, Weak Duality Theorem, Wolfe's Duality Theorem, Hanson-Huard strict converse duality theorem, Dorn's duality theorem, strict converse duality theorem, Dorn's Converse duality theorem, Unbounded dual theorem, theorem on no primal minimum .Duality in Quadratic Programming.

Quadratic programming: Wolfe's method, Beale's method for Quadratic programming.

Linear fractional programming, method due to Charnes and Cooper. Nonlinear fractional programming, Dinkelbach's approach.

Game theory - Two-person, Zero-sum Games with mixed strategies, graphical solution, solution by Linear Programming.

[Scope as in Chapter 17 of reference no.4, Chapter 3 & 4 of reference no.1, chapter 24, 26 and 28 of reference no.2, Chapter 8 of reference no.3, chapter 16 of reference no.5]

**References:**

1. Mokhtar S. Bazaraa & C.M. Shetty: Nonlinear Programming, Theory & Algorithms, 2<sup>nd</sup> Edition, Wiley, New-York, 2004.
2. S.M. Sinha: Mathematical Programming, Theory and Methods, Elsevier, 1<sup>st</sup> Edition, 2006.
3. O. L. Mangasarian: Nonlinear Programming, TATA McGraw Hill Company Ltd. (Bombay, New Delhi), 1<sup>st</sup> Edition, 1969.
4. Kanti Swarup, P.K. Gupta & Man Mohan: Operations Research Sultan Chand & Sons, New Delhi 9<sup>th</sup> Edition, 2001
5. N.S. Kambo, Mathematical Programming Techniques, Affiliated East-West Press Pvt. Ltd. New Delhi, Madras, 1984, Revised Edition, Reprint 2005.

**MATH 770S: Functional Analysis**

[7 hrs per week (including tutorials)]

Max.Marks : 100

[Final-80+Internal Assessment-20]

Time: 3hrs.

**Objective**

*This course is an introduction to Banach Spaces and Hilbert Spaces along with various operators/functionals so as to enable the students to study advanced topics in Functional Analysis like Spectral theory, Topological Vector Spaces, Banach Algebras etc., Baire' Category theorem and its applications are also dealt with.*

**Note : 1.The question paper will have eight questions. Candidates will attempt five questions.**

**2. There will be four questions from each part and the students will be required to answer at least two questions from each part.**

**3. All questions carry equal marks.**

**PART- I**

Baire Category theorem and its applications.

[Scope as in relevant topics of Chapter I from Ref.2]

Normed Spaces, with examples of Function spaces  $L^p([a,b])$ ,  $C([a,b])$  and  $C^1([a,b])$ , Sequence Spaces  $l^p$ ,  $c$ ,  $c_0$ ,  $c_{00}$  Banach Spaces Hahn Banach theorem, open mapping theorem, closed graph theorem, Banach Steinhaus theorem (uniform boundedness principle), [Scope as in relevant topics from Chapter 2 & 3 of Ref.6.]

**PART- II**

Geometry of Hilbert spaces: Inner product spaces, orthonormal sets, Approximation and optimization, Projections and Riesz Representation theorem.

Bounded Operators on Hilbert spaces: Bounded operators and adjoints; normal, unitary and self adjoint operators, Spectrum and Numerical Range.

[Scope as in Ch.VI & VII (§25-27.7) of the book 'Functional Analysis' by B.V.Limaye, 1996.]

**References:**

1. S.K. Berberia: Introduction to Hilbert Spaces, (N.Y. O.W.P.), 1<sup>st</sup> Edition, 1961.
2. C. Goffman and G. Pedrick: First course in Functional Analysis, N.Delhi Prentice Hall of India), 1<sup>st</sup> Edition, 1965.
3. F.K. Riesz and Bela Sz Nagy: Functional Analysis, (N.Y., Wingar), 1955.
4. A.H.Siddiqui: Functional Analysis (Tata-McGraw Hill), 1986.
5. Walter Rudin : Real and Complex Analysis(McGraw-Hill) 3<sup>rd</sup> Edition, 1987.

6. B.V. Limaye: Functional Analysis (Wiley Eastern Ltd.), 2<sup>nd</sup> Edition, 1985.
7. Royden, H.L.: Real Analysis, Pearson Prentice Hall, Dorling Kindersley (P) Ltd India, Third Edition 1988.

**Math 769S: Commutative Algebra II****[7 hrs per week (including tutorials)]****Max.Marks : 100****[Final-80+Internal Assessment-20]****Time: 3hrs.****Objective**

*Commutative Algebra is the study of commutative rings, their modules and ideals. This theory has been developed over the last 150 years not just as an area of algebra considered for its own sake, but as a tool in the study of two enormously important branches of mathematics: algebraic geometry and algebraic number theory. This course will give the student a background in commutative algebra which is used in both algebraic geometry and number theory.*

- Note :**
- 1. The question paper will have eight questions. Candidates will attempt five questions.**
  - 2. There will be four questions from each part and the students will be required to answer atleast two questions from each part.**
  - 3. All questions carry equal marks.**

**PART-I**

Rings and modules of fractions, local properties, extended and contracted ideals in rings of fraction, primary decompositions, integral dependence, the going up theorem, integrally closed domains, the going down theorem, valuation rings.

**PART-II**

Chain conditions, Noetherian and Artinian modules, Noetherian rings, primary decomposition in Noetherian rings, Noether-Lasker Theorem.

**Suggested Books**

1. M. F. Atiyah and I. G. Macdonald, Introduction to Commutative Algebra, Levant Books, Indian Edition, 2007.
2. O. Zariski and P. Samuel, **Commutative Algebra**, (vols I and II). Springer 1975.
3. Gopalakrishnan, N.S., Commutative Algebra, Oxonian Press (New Delhi) 1984
4. D.S. Dummit and R. M. Foote, Abstract Algebra, 2nd Ed., John Wiley, 2002.
5. N. Jacobson, Basic Algebra II, 2nd Ed., W. H. Freeman, 1980.
6. S. Lang, Algebra, 3rd Ed., Springer (India), 2004.
7. R. Y. Sharp: Steps in Commutative Algebra, London Math. Soc. Student Text 19, Cambridge University Press, 1990.

## MATH 781S: Computational Techniques –II

### Theory

[4 hrs per week (including tutorials)]

Max. Marks: 80

[Final-60+Internal Assessment-20]

Time: 3hrs.

### **Objective**

*The aim of this course is to learn the basics of computer program in Programming in 'C' at their own. For the purpose of learning programming skill, some Numerical methods which are extremely useful in scientific research are included. For practising the programmes of the numerical method, the course of practical has also been included in this paper. The contents of the curriculum has been developed keeping in view the UGC guidelines.*

- Note :**
- 1. The question paper will have eight questions. Candidates will attempt five questions.**
  - 2. There will be four questions from each part and the students will be required to answer at least two questions from each part.**
  - 3. All questions carry equal marks.**
  - 4. Use of scientific calculator is allowed for numerical work.**

### **PART – I**

**Programming in C:** Historical development of C, Character set, constants, variables, C-key words, Instructions, Hierarchy of operations, Operators, Simple C programs, Control structures: The if, if-else, nested if-else, unconditional goto, switch structure, Logical and conditional operators, while, do-while and for loops, Break and continue statements, Arrays, Functions, recursion, Introduction to pointers.

### **PART – II**

**Curve fitting:** Linear and non-linear curve fitting, curve fitting by sum of exponentials, fitting of exponential and trigonometric functions.

**Solution of Linear system of equations:** Matrix inversion, Gauss-elimination and Gauss-Jordan method, LU decomposition method, Gauss Seidal method.

**Solution of differential equations:** Taylor's series, Euler's, Modified Euler's, Runge –Kutta methods and their extensions, Stability Analysis, Predictor – Corrector methods, Finite Difference and Shooting methods to solve BVP, FDM for Laplace and Heat equations.



## **Computational Techniques (Practical)-II**

**[3 hrs per week, Max. Marks: 20]\***

Writing programs in C for the problems based on the method studied in theory paper and run them on PC.

Practical shall be conducted by the department as per the following distribution of marks:

Writing program in FORTRAN and running it on PC=10 Marks

Practical record=5 Marks

Viva-Voice=5 Marks

### **References:**

1. C. Xavier:C Language and Numerical Methods, New Age Int. Ltd., 2007.
2. Y. Kanetkar: Let us C, BPB Publications, 2007.
3. V Raja Raman: Computer oriented Numerical Methods, PHI, 2001.
4. S. S. Shastry: Introductory Methods of Numerical Analysis, PHI, 2005
5. C. F. Gerald and P. O. Wheatley: Applied Numerical Analysis, Pearson Education, Asia, 2004.

**\*NOTE: There will be no internal assessment in the Practical examination.**

**MATH: 791S: Algebraic Number Theory-II**

[7 hrs per week (including tutorials)]

**Max.Marks : 100****[Final-80+Internal Assessment-20]****Time: 3hrs.****Objective**

*This is a continuation of the course Math 771S which was introduction to algebraic number theory. In the present course some advanced topics of algebraic number theory are studied.*

- Note :**
- 1. The question paper will have eight questions. Candidates will attempt five questions.**
  - 2. There will be four questions from each part and the students will be required to answer atleast two questions from each part.**
  - 3. All questions carry equal marks.**

**PART-I**

Relative extensions, index of ramification, residual degree, Fundamental equality. Different and relative discriminant, Dedekind's theorem on ramified primes.

**PART-II**

Finiteness of class number, Determination of class numbers in special cases, Dirichlet's class number formula and simple applications.

**References**

1. D.A.Marcus: Number Fields, Sprinder-Verlag, New York, 1977.
2. R.A.Mollin: Algebraic Number theory, Chapman & Hall/CRC, 2011.
3. P. Samuel: Algebraic Theory of Numbers, Dover Publications, 1970.
4. P. Ribenboim: Classical Theory of Algebraic Number, Sprinder-Verlag, New York, 2001.
5. I. Stewart and D. Tall: Algebraic Number theory, 2<sup>nd</sup> Edition, Chapman & Hall, 1997.

**MATH: 792S: Topics in Number Theory –II**

[7 hrs per week (including tutorials)]

**Max.Marks : 100****[Final-80+Internal Assessment-20]****Time: 3hrs.****Objective**

*Objective of this course is to illustrate further applications of basic series in the study of many combinatorial objects like Stirling numbers, Catalan Numbers, Ramanujan's mock theta functions and Agarwal-Andrews coloured partitions.*

- Note :**
- 1. The question paper will have eight questions. Candidates will attempt five questions.**
  - 2. There will be four questions from each part and the students will be required to answer atleast two questions from each part.**
  - 3. All questions carry equal marks.**

**PART-1**

Fibonacci numbers and their connection with partitions, Andrews' polynomial identity which implies Rogers-Ramanujan identities, r-Fibonacci sets and their applications in combinatorics, q-Fibonacci numbers. Recurrence relations, generating functions and other combinatorial properties of Stirling and q-Stirling numbers of the First and Second kinds. Bernoulli numbers and their connection with Reimann zeta function.

**PART - II**

Properties and applications of Catalan and q-Catalan numbers in combinatorics. 'Rank' and 'Crank' of a partition and their applications in providing combinatorial interpretations of the Ramanujan congruences. Mock theta functions and their combinatorial interpretations. n-Colour partitions, Conjugate and self-conjugate n-colour partitions, Restricted n-colour partitions, Rogers-Ramanujan type identities for n-colour partitions.

**Suggested Readings**

1. A.K. Agarwal, Padmavathamma and M.V. Subbarao, Partition Theory, Atma Ram & Sons, Chandigarh, 2005.
2. R.P. Agarwal, Resonance of Ramanujan Mathematics, Vol.2 (New Age International), 1996.
3. H. Gupta, Selected Topics in Number Theory, ABACUS Press, 1980.
4. N.J. Fine, Basic Hypergeometric Series and Applications, Mathematical Surveys and Monographs, No. 27, American Mathematical Society, 1988.
5. J. Riordan, An Introduction to Combinatorial Analysis, Princeton University Press, 1978.
6. L. Comtet, Advanced Combinatorics, D. Reidel Publishing Company, 1974.

**MATH-793S: Fluid Mechanics –II**

[7 hrs per week (including tutorials)]

Max.Marks : 100

[Final-80+Internal Assessment-20]

Time: 3hrs.

**Objective**

*This course is designed to make the students learn to develop mathematical models of fluid dynamical systems and use mathematical techniques to find solutions to these models.*

- Note :**
- 1. The question paper will have eight questions. Candidates will attempt five questions.**
  - 2. There will be four questions from each part and the students will be required to answer atleast two questions from each part.**
  - 3. All questions carry equal marks.**

**PART-I**

**Viscous Flows:** Stress components in a real fluid, Relation between Cartesian components of Stress, The rate of Strain quadric and principle stresses, The Coefficient of Viscosity and laminar flow, The Navier Stokes equation of motion and applications, plane Poiseuille flows.

Flow through tubes of uniform cross section in the form of circle, Ellipse, equilateral triangle, under constant pressure gradient. Diffusion of vorticity. Energy dissipation due to viscosity, steady flow past a fixed sphere, dimensional analysis, Reynold numbers, Prandtl's boundry layer. Boundary layer equation in two dimensions, Karman integral equation.

**PART-II**

**Wave theory:** Elements of wave motion, waves in fluids, Surface gravity waves, standing waves, group velocity, energy of propagations, path of particles, waves at interface of two liquids.

**Magnetohydrodynamics:** Maxwell's electromagnetic field equations, the equation of motion of a conducting fluid, rate of flow of charge, the magnetic Reynold number, Alfve'n s theorem, The magnetic body force, Ferrero's law of isorotation, magnetohydrostatics, magnetohydrodynamic waves, laminar flow in transverse magnetic field.

**References**

1. Chorlton, F. :Text Book of Fluid Dynamics, CBS Publishers, Indian Edition, 2004.
2. L.D.Landau & E. N. Lipschitz: Fluid Mechanics, 2<sup>nd</sup> Edition, Vol. 6 (Course of Theoretical Physics), Pergamon Press Ltd., 1987.
3. G. K. Batchelor: An Introduction to Fluid Mechanics, Cambridge University Press, 1967.
4. Kundu and Cohen: Fluid Mechanics, Indian Reprint, Published by Harcourt (India) Pvt.Ltd., 2003.

## Math 794S:Algebraic Coding Theory-II

[7 hrs per week (including tutorials)]

Max.Marks : 100

[Final-80+Internal Assessment-20]

Time: 3hrs.

### Objective

*Cyclic codes play a significant role in the theory of error correcting codes. They can be efficiently encoded using shift registers, which explains their preferred role in engineering. The objectives of this course is to teach the algebraic structure of cyclic codes over fields and rings, their properties and some special cyclic codes.*

- Note :**
1. **The question paper will have eight questions. Candidates will attempt five questions.**
  2. **There will be four questions from each part and the students will be required to answer atleast two questions from each part.**
  3. **All questions carry equal marks.**

### PART-I

Review of finite fields, Factorization of  $x^n-1$  over finite fields.

Subfield codes, Concatenated Codes, Trace Codes, Cyclic codes, decoding of cyclic codes, idempotents and multipliers, zeros of a cyclic code, decoding of cyclic codes, minimal cyclic codes, BCH codes.

### PART-II

Duadic codes, Orthogonality of Duadic codes, Weights in Duadic codes, Quadratic Residue codes, Reed-Soloman codes, Generalized Reed Soloman codes, Cyclic codes over rings specially over  $Z_4$ .

### References

- 1 San Ling and Chaoping Xing- Coding Theory, Cambridge University Press, 1<sup>st</sup> Edition, 2004.
- 2 W. C. Huffman and Vera Pless – Fundamentals of Error Correcting Codes, Cambridge University Press, 1<sup>st</sup> South Asian Edition, 2004.
- 3 Raymond Hill- Introduction to Error Correcting Codes, Oxford University Press, 1986, reprint 2009.
- 4 F. J. MacWilliams and N.J.A.Sloane – Theory of Error Correcting Codes Part I & II, Elsevier/North-Holland, Amsterdam, 1977.
- 5 Vera Pless - Introduction to Theory of Error Correcting Codes, Wiley-Interscience, 3<sup>rd</sup> Edition, 1982.

## Math 795S-Representation Theory of Finite Groups

[7 hrs/per week (including Tutorials)]

[Max. Marks: 100]

(Final-80+Internal Assessment-20)

Time : 3hrs

### Objective

The representation theory of finite groups solidifies one's knowledge of group theory. It goes back to F. Klein who considered the possibility of representing a given abstract group by a group of linear transformations (matrices) preserving the group's structure. Leading mathematicians such as G. Frobenius, I. Schur, W. Burnside and H. Maschke followed and developed the idea further. Essentially, it is designed to give an explicit answer to the question "What are the different ways (homomorphisms) a finite group  $G$  can occur as a group of invertible matrices over a particular field  $F$ ?". The link between group representations over a field  $F$  and modules is obtained using the concept of a group ring  $F[G]$ , thus an essential step is the systematic study and classification of group rings (the so-called semisimple algebras) which behave like products of matrix rings. Therefore, the story of the representation theory of a group is the theory of all  $F[G]$ -modules, viz modules over the group ring of  $G$  over  $F$ . The ultimate goal of this course is to teach students how to construct complex representations for popular groups as well as their character tables which serve as invariants for group rings. In addition to the applications to physical symmetry, the theory leads to significant applications to the structure theory of finite groups.

- Note :**
- 1. The question paper will have eight questions. Candidates will attempt five questions.**
  - 2. There will be four questions from each part and the students will be required to answer atleast two questions from each part.**
  - 3. All questions carry equal marks.**

### PART-I

The Semisimplicity of Group Algebras, Maschke's Theorem, Examples of Decompositions of Group Algebras, Simple Modules over  $K[G]$ , Cyclic Modules over  $K[G]$ , Representations, Characters of Representations, Group Characters, Orthogonality relations, Ordinary and Modular Representations, Examples of Representations.

### PART-II

Some more examples of rational group representations, Integrity of Complex Characters, Burn Burnside's  $p^a q^b$ -Theorem, Tensor product of representations, Induced representations, Restriction and induction, Frobenius reciprocity theorems, Conjugate representations, Clifford's decomposition theorem, Mackey's irreducibility criteria,

## References

1. I. N. Herstein, Non-Commutative Rings, The Carus Mathematical Monograph, The Mathematical Association of America, 1968.
2. C. Musili, Representations of Finite Groups, Hindustan Book Agency, 1993.
3. J. -P. Serre, Linear Representations of Finite Groups, Graduate Texts in Mathematics; 42, Springer Verlag, 1977.
4. T. Y. Lam, A First Course in Non-commutative Rings, Graduate Texts in Mathematics; 131, Springer Verlag, 1991.
5. N. Jacobson, Basic Algebra II, 2nd Ed., W. H. Freeman, 1980.

## MATH 796S: Partial Differential Equations –II

[7 hrs/ per week (including Tutorials)]

[Max. Marks: 100]

(Final-80+Internal Assessment-20)

Time : 3hrs

### Objective

*The objective of this course is to enable the students to understand the concepts related to the solution of partial differential equations in arising in various fields.*

- Note :**
1. **The question paper will have eight questions. Candidates will attempt five questions.**
  2. **There will be four questions from each part and the students will be required to answer atleast two questions from each part.**
  3. **All questions carry equal marks.**

### PART-I

Weak solutions, Uniqueness, Conservation Laws: Shocks, entropy condition, Lax-Oleinik formula, Weak solutions, Uniqueness, Riemann's Problem, Long time behavior.

Representation of solution-Separation of variables, Similarity solutions – Plane and traveling waves, Solitons, Similarity under scaling.

### PART-II

Transform Methods- Laplace and Fourier transform, Hopf-Cole Transformation, Potential functions, Hodograph and Legendre transform, Asymptotics- Singular perturbations, Laplace Method, Geometric optics, Stationary Phase, Homogenization, Power Series- Non-Characteristic surfaces, Real Analytic functions, Cauchy-Kovalevskaya Theorem.

### References:

1. L. C. Evans: Partial Differential equations, Graduate Studies in Mathematics Vol 19, American Mathematical Society, (1998).
2. Robert C. McOwen: Partial Differential Equations methods and applications, 2<sup>nd</sup> Edition, Pearson Education Inc., 2003.

3. H.F.Weinbergergerger: A first course in Partial Differential Equations with complex variables and transform methods. Corrected reprint of the 1965 original, dover Publications, Inc., NY, 1995.
4. P.Prasad and R.Ravindran, Partial differential equations. New Age International, 2012.

### **MATH 797S: Continuum Mechanics-II**

**[7 hrs/per week (including Tutorials)]**

**[Max. Marks: 100]**

**(Final-80+Internal Assessment-20)**

**Time : 3hrs**

#### **Objective**

This course is in continuation of Math775S: Continuum Mechanics-I. The aim of this course is to acquaint the students about the field equations and constitutive relations for isotropic bodies. To understand the dynamical behaviour of elastic bodies, some basic problems are included on wave propagation.

- Note :**
- 1. The question paper will have eight questions. Candidates will attempt five questions.**
  - 2. There will be four questions from each part and the students will be required to answer atleast two questions from each part.**
  - 3. All questions carry equal marks.**

#### **PART – I**

**Equations of Elasticity:** Strain energy density function, generalized Hooke's law, Elastic constants and their significance, Homogeneous isotropic media, Displacement equation of motion for uniform media, uniqueness of solution, Beltrami-Michel compatibility equation.

**Thermo-elasticity:** Thermal stresses, Duhamel-Numann law, Dynamical equations of thermoelastic problems.

#### **PART – II**

Thermal stresses in spherical bodies.

Two dimensional propagation of elastic waves in isotropic solid, waves of dilatational and waves of distortion, Reflection of P, SV and SH waves from free surface of an elastic half-space, Reflection and refraction of these waves from solid-solid interface, Surface waves-Rayleigh and Love waves.

#### **References:**

1. Sokolnikoff, I. S.: Mathematical theory of elasticity, Mc-Graw-Hill, 2<sup>nd</sup> Edition, 1982.



2. **Ewing** W. M. **Jardetzky** W. S. **Press** F. 1957. Elastic Waves in Layered Media. McGraw-Hill Book Co.

## **MATH 798S : Numerical Methods for Differential Equations –II**

### **Theory**

[4 hrs per week theory (including tutorials)]

Max. Marks: 80

[Final-60+Internal Assessment-20]

Time: 3hrs.

### **Objective**

At the end of the course, the students will be able to understand the basic concepts in the Numerical Analysis of Partial Differential Equations and use to MATLAB to compute the numerical solution of the Partial Differential Equations.

### **Note:**

1. **The question paper will have eight questions. Candidates will attempt five questions.**
2. **There will be four questions from each part and the students will be required to answer at least two questions from each unit.**
3. **All questions carry equal marks.**

### **PART-I**

**An Overview:** Classification, Fourier's Method, Integral Transform, Maximum Principle, Fundamental Solution.

### **Parabolic Initial-Boundary Value Problems:**

Parabolic equations in one space variable: A model problem, series approximation, an explicit scheme for the model problem, truncation error, convergence of the explicit scheme, Fourier analysis of the error, an implicit method, the Thomas algorithm, the weighted average or  $\theta$ -method, a maximum principle and convergence for  $\mu(1-\theta) \leq \frac{1}{2}$ , a three-time level scheme, more general linear problems, nonlinear problems.

Parabolic problems in two and three space variables: The explicit method in a rectilinear box, an ADI method in two dimensions, ADI and LOD methods in three dimensions, curved boundaries, application to general parabolic problems.

### **PART-II**

**Hyperbolic Equations:** Boundary conditions, convergence, consistency, stability, The Courant-Friedrichs-Lewy condition. Fourier and Von Neumann

Analysis of finite difference schemes, Lax-Richtmyer equivalence theorem, Stability of the Lax-Wendroff, Crank-Nicolson, finite volume schemes, Leapfrog and general multistep schemes, the theory of Schur and Von Neumann polynomials, the algorithm for Schur and Von Neumann polynomials.

**Elliptic Boundary Value Problems:** A model problem, Error analysis of the model problem, the general diffusion equation, boundary conditions on a curved boundary, error analysis using a maximum principle, asymptotic error estimates.

## Practical

[3 hrs per week, Max. Marks: 20]  
Time: 3hrs

Writing programs in Matlab for the following problems and run them on PC.

Consider the heat equation

$$u_t - u_{xx} = 0, \quad 0 < x < \pi, \quad 0 < t$$

with the initial data

$$u(x, 0) = \sin x$$

with the boundary data

$$u(0, t) = 0, \quad u(\pi, t) = 0.$$

Write program in Matlab to find the numerical solution of the initial-boundary value problem

1. Using Forward in time and central in space (FT-CS).
2. Using Backward in time and central in space (BT-CS).
3. Using Crank-Nicolson scheme (CNS).
4. Compare all the above schemes via plotting the numerical solution in Matlab.

Consider the initial-boundary value problem

$$u_t + u_x = 0, \quad \text{on } -2 < x < 3, \quad 0 < t$$

with the initial data

$$u(x, 0) = \begin{cases} 1 - |x| & \text{if } |x| \leq 1 \\ 0 & \text{if } |x| > 1 \end{cases}$$

with the boundary data

$$u(-2, t) = 0, \quad t \geq 0.$$

Write program in Matlab to find numerical solution of the problem

5. Using Forward-time backward-space.
6. Using Forward-time central-space scheme.
7. Using Lax-Friedrichs scheme.
8. Using Leapfrog scheme.
9. Compare all the above schemes via plotting the numerical solution in Matlab.

Consider the Burger's equation

$$u_t + \left(\frac{u^2}{2}\right)_x = 0, \quad 0 \leq x \leq 1, \quad 0 \leq t$$

with the initial data

$$u(x, 0) = \exp[-10(4x - 1)^2], \quad 0 \leq x \leq 1$$

with the boundary data

$$u(0, t) = 0, \quad t \geq 0.$$

10. Write a program to find the Numerical solution using Lax-Wendoff method.

Consider the Laplace equation

$$u_{xx} + u_{yy} = 0, \quad (x, y) \in (0, 1) \times (0, 1)$$

with the boundary conditions

$$\begin{aligned} u(x, 0) &= 0, & u(x, 1) &= x \\ u(0, y) &= 0, & u(1, y) &= y. \end{aligned}$$

11. Write a program in Matlab to find the numerical solution of the above problem.
12. Compare the numerical solution with exact solution.
13. Find the absolute and relative Error in maximum and  $L^2$  norms.

### References:

1. K.W. Morton and David Mayer, Numerical Solution of Partial Differential Equations, Cambridge University Press, Cambridge, 2<sup>nd</sup> Edition, 2005.
2. J.C. Strikwerda, Finite difference Schemes and Partial Differential Equations, Second Edition, SIAM, Philadelphia, 2<sup>nd</sup> Edition, 2004.
3. R.J. Leveque, Finite Difference Methods for Ordinary and Partial Differential Equation: Steady-State and Time-Dependent Problems, SIAM, Philadelphia, 2007.
4. L.C. Evans, Partial Differential Equations, 2<sup>nd</sup> Edition, AMS Press, 2010.

5. C. Grossmann, H.G. Roos and M. Stynes, Numerical Treatment of Partial Differential Equations, Springer Verlag, Berlin Heidelberg, 2007.
6. J.W. Thomas, Numerical Partial Differential Equations : Finite Difference Methods, Texts in Applied Mathematics, Vol. 22, Springer Verlag, New York, 1995.
7. J.W. Thomas, Numerical Partial Differential Equations: Conservation Laws and Elliptic Equations, Texts in Applied Mathematics, Vol. 33, Springer Verlag, New York, 1999.
8. G.D. Smith, Numerical Solutions of Partial Differential Equations : Finite Difference Methods , 3rd Edition, Oxford University Press, New York, 1985.

### **Math 799S: Numerical optimization**

#### **Theory**

**[4 hrs per week theory (including tutorials)]**

**Max. Marks: 80**

**[Final-60+Internal Assessment-20]**

**Time: 3hrs.**

#### **Objective**

To acquaint the students with optimization techniques for solving nonlinear programming problems and discuss their theoretical base and to apply these techniques on various problems using MATLAB on computers.

- Note:**
- 1. The question paper will have eight questions. Candidate will attempt five questions.**
  - 2. There will be four questions from each part and students will be required to answer at least two questions from each part.**
  - 3. All questions carry equal marks.**
  - 4. Use of scientific calculators is allowed for numerical work.**

### **PART-1**

Algorithms and Algorithmic maps, closed maps and convergence, composition of mappings. Unconstrained optimization: line search without using derivatives, uniform search , dichotomous search, The Golden Section method, Fibonacci method. Line search using derivatives, Bisection method, Newton's method and their convergence. Multi dimensional search without using derivatives, the Method of Rosenbrock, Cyclic coordinate method, Method of Hooke and Jeeves and their convergence.

Multidimensional search using derivatives, Steepest Descent algorithm and its convergence analysis, Newton's method and modified Newton's method. Methods using conjugate directions: the method of Davidon-Fletcher-Powell (DFP) method, the Broyden-Fletcher-Goldfarb-Shanno (BFGS) method, Conjugate Gradient method of Fletcher and Reeves and their convergence, subgradient optimization.

Scope as in chapter 7,8 of Ref. No.1.

## **PART-II**

Constrained optimization: Indirect methods, the concept of penalty functions, exterior penalty function method (EPF), exact absolute value and augmented Lagrangian Penalty methods and their convergence analysis. Direct methods, successive linear programming approximation (SLP), successive quadratic programming approximation (SQP), gradient project method of Rosen, generalized reduced gradient method (GRG), convex simplex algorithm of Zangwill. Scope as in chapter 9 and chapter 10 of Ref. No.1.

### **Practical**

**[3 hrs per week, Max. Marks:20]\***

Writing programs in Matlab for the problems based on methods studied in theory paper and run them on PC.

Practical shall be conducted by the department as per the following distribution of marks.

Writing program on Matlab and  
running on PC=10 marks

Practical record=5 marks

Viva-Voice=5 marks

**\*NOTE: There will be no internal assessment in the Practical examination.**

### **References**

1. Mokhtar S. Bazaraa, Hanif D. Sherali and C.M Shetty, Nonlinear Programming, Theory and Algorithms, John Wiley and Sons, 2004.

2. P.Venkataraman, Applied Optimization with MATLAB Programming, John Wiley and Sons, 2009.

### **Math 800S: Measure and Integration-II**

**[7 hrs per week (including tutorials)]**

**Max.Marks : 100**

**[Final-80+Internal Assessment-20]**

**Time: 3hrs.**

#### **Objective**

The objective of this course is to study differentiation of measures, Fubini's Theorem in a general setting. Also some elementary results on Banach algebras are studied.

- Note :**
- 1. The question paper will have eight questions. Candidates will attempt five questions.**
  - 2. There will be four questions from each part and the students will be required to answer atleast two questions from each part.**
  - 3. All questions carry equal marks.**

#### **PART I**

Differentiation: Derivatives of measures, the Fundamental theorem of Calculus, Differentiable transformations.

Integration on product spaces: Measurability on Cartesian products, product measures, the Fubini Theorem, completion of product measures, convolutions, distribution functions

#### **PART-II**

Fourier transforms, formal properties, the inversion theorem, the Plancherel theorem, the Banach algebra  $L^1$ .

Elementary theory of Banach algebras, The invertible elements, ideals and homomorphisms, applications.

Holomorphic Fourier transforms; two theorems of Paley and Weiner.

(Scope as in Chapters 7, 8, 9 18, & 19 of the book at reference No.4)

#### **References:**

1. E. Hewitt and K. A. Ross: Abstract Harmonic Analysis, Springer Verlag , Berlin, Vol. I , 1963 , Vol. II, 1970.

2. C. E. Rickart: General theory of Banach algebras, D. Van Nostrand Company, Inc.. Princeton N. J. 1960
3. H. L. Royden: Real Analysis, third edition , Pearson Prentice Hall, 2009.
4. W. Rudin: Real and Complex Analysis, third edition TMH, New Delhi, 2010.

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